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Overview



密碼學與應用 海洋大學資訊工程系 丁培毅

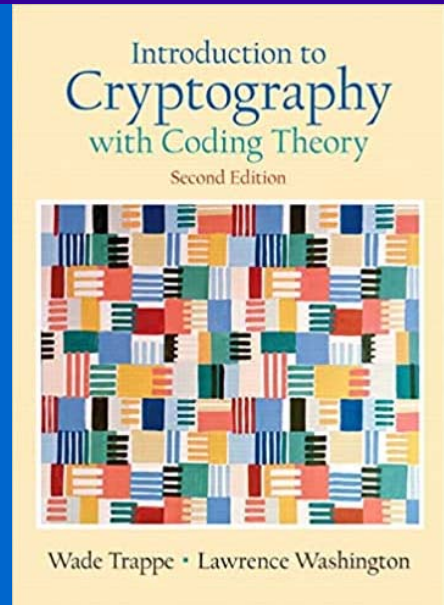
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Course Information

Course materials:

<http://squall.cs.ntou.edu.tw/CryptoIntro/>

<http://tronclass.ntou.edu.tw/>



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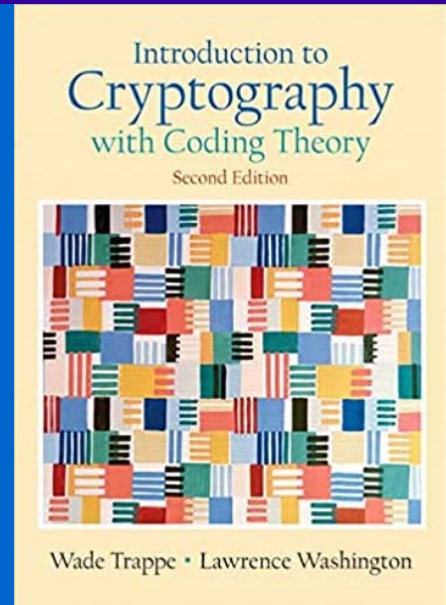
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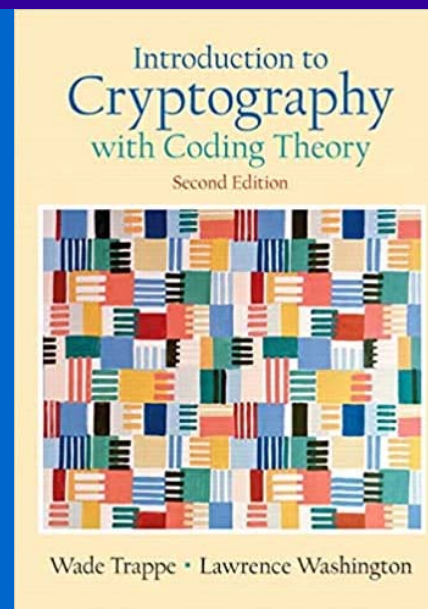
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Basic course contents:

Fundamental cryptography and its applications in constructing **secure information infrastructure**: networking environments, distributed computing resources, cloud services, and edge computing facilities.

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Overview of Cryptography

- People want **privacy**, **security (confidentiality, integrity, authenticity, and availability)**, and **safety** while communicating

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- In the past, cryptography is heavily used for military applications to keep sensitive information secret from enemies (adversaries).

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 - **Julius Caesar** used a simple **shift cipher** to communicate with his generals in the battlefield.

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Example: Let the key $k = 17$

- In
ap
fro
Plaintext: $X = \text{a t t a c k} = (0, 19, 19, 0, 2, 10)$
Ciphertext : $Y = (0+17 \bmod 26, 19+17 \bmod 26, \dots)$
 $\quad \quad \quad = (-17, 10, 10, 17, 19, 1) = \text{R K K R T B}$

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 - **Julius Caesar** used a simple **shift cipher** to communicate with his generals in the battlefield.
 - World War I, World War II (Enigma)

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- Nowadays, with the fast technologic progress, our dependency on computer systems and networks has increased a lot such that we need **more sophisticated** techniques to ensure the smooth operations.

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- Nowadays, with the fast technologic progress, our dependency on computer systems and networks has increased a lot such that we need **more sophisticated** techniques to ensure the smooth operations.
- **Cryptography** provides most of the methods and techniques for **secure communication** and **secure computing**

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- The meaning of **privacy** depends on its context: legal, political, societal, cultural, and sociotechnological

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 6. intimacy – controlled access to one's intimate aspects of life

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Privacy vs. **National Security**

Should privacy be sacrificed for security (safety)?

4. control over personal information
5. the protection of one's personality, individuality and dignity
6. intimacy – controlled access to one's intimate aspects of life

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Terminology

- **Cryptology**: A term used for the study of secure mechanisms for communication over **insecure channels** and related problems.
- **Cryptography**: The process of designing systems to realize secure communications over insecure channels.
- **Cryptoanalysis**: The discipline of breaking cryptographic systems.

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Terminology

- **Coding Theory:** Deals with representing the information using codes. It covers: compression, secrecy, and error-correction.
 - Recently, it is predominantly associated with error-correcting
 - codes which ensures the correct transmissions over **noisy-channels**.

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The Aspects of Cryptography

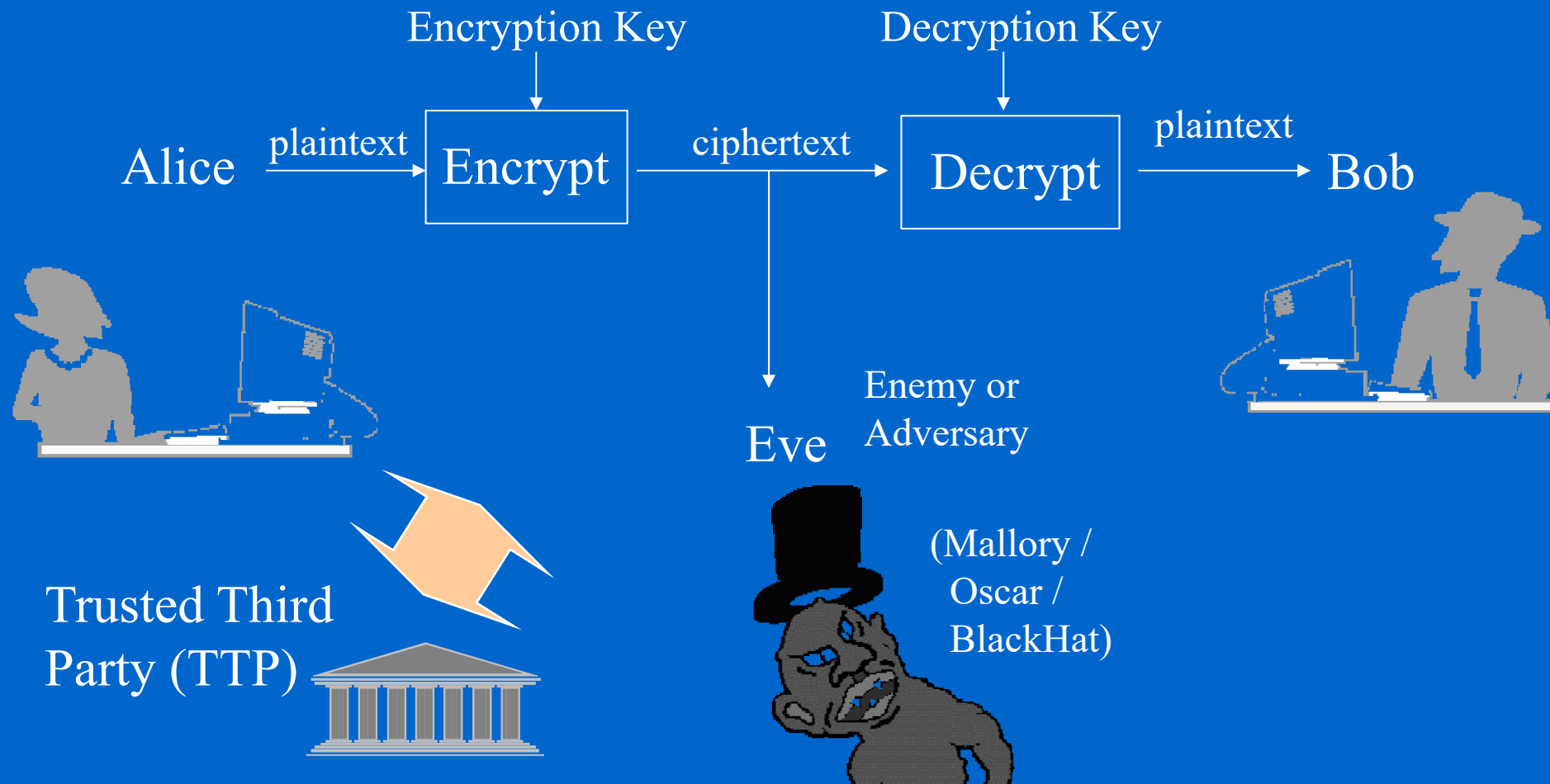
- Modern cryptography heavily depends on mathematics and the usage of digital systems.
- It is an inter-disciplinary study of basically three fields:
 - Mathematics
 - Computer Science
 - Electrical Engineering

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The Aspects of Cryptography

- Without having a complete understanding of cryptanalysis / cryptanalytic techniques / provable security, it is impossible to design *good* (secure, unbreakable) cryptographic systems.
- It makes use of other disciplines such as number theory, quantum physics, error-correcting codes, and computation theory.

Basic Communication Scenario



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Goal of Cryptographic Systems

- Confidentiality
- Authentication
- Integrity
- Non-repudiation
- Access control (Identification)

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Eve's Goals

- (1) Peep the transmitted message.
- (2) Figure out the key Alice is using and read all the messages encrypted with that key.
- (3) Modify the content of the message in such a way that Bob will think Alice sent the corrupted message.
- (4) Impersonate Alice and communicate with Bob who thinks he is communicating with Alice.

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Eve's Goals (cont'd)

- Eve or Oscar is a **passive** observer who tries to perform (1) and (2).
- Mallory is more **active** and evil who tries to perform (3) and (4).

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Network Security Attacks

Security attack: any action that compromises the security of information

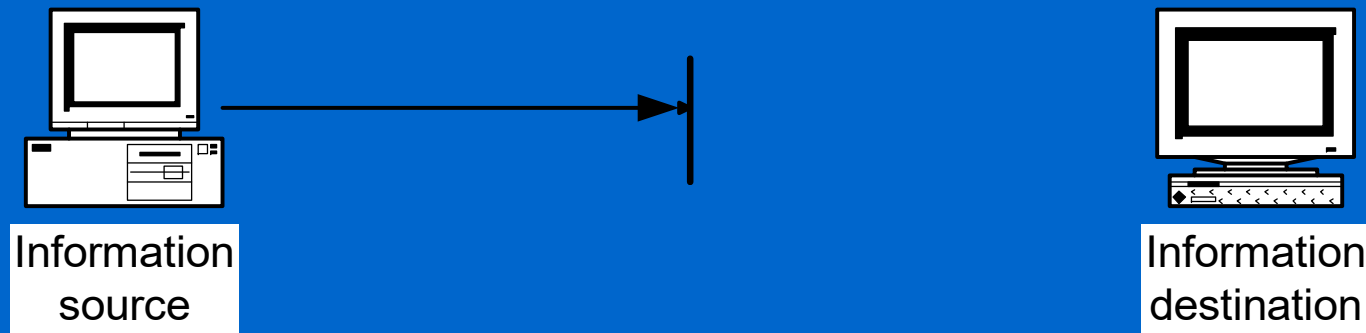
Four general categories of attacks: [W. Stalling]

- Interruption
- Interception
- Modification
- Fabrication

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Interruption

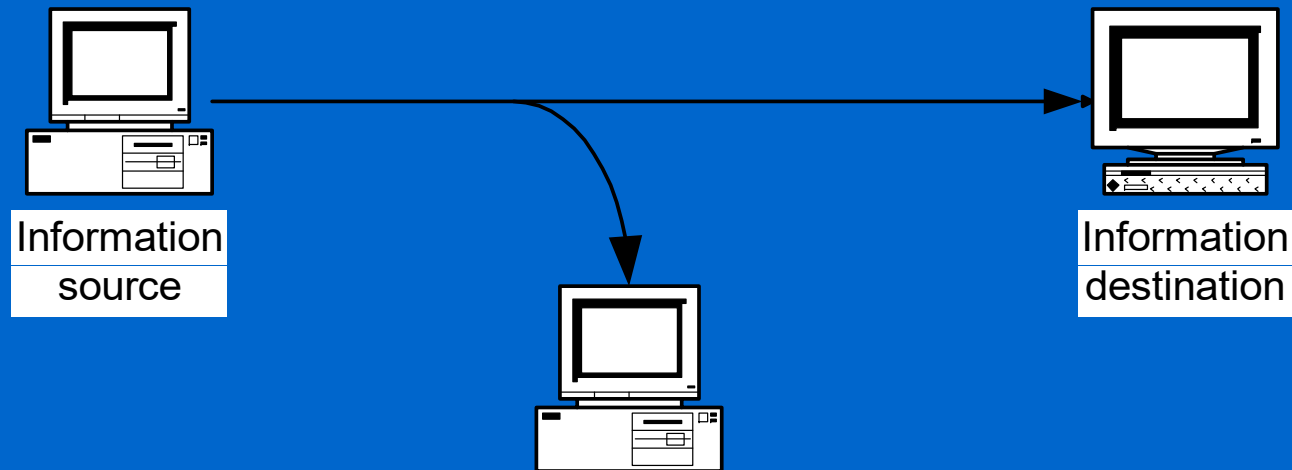
- An asset of the system is destroyed or becomes unavailable or unusable
- This is an attack on **availability**



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Interception

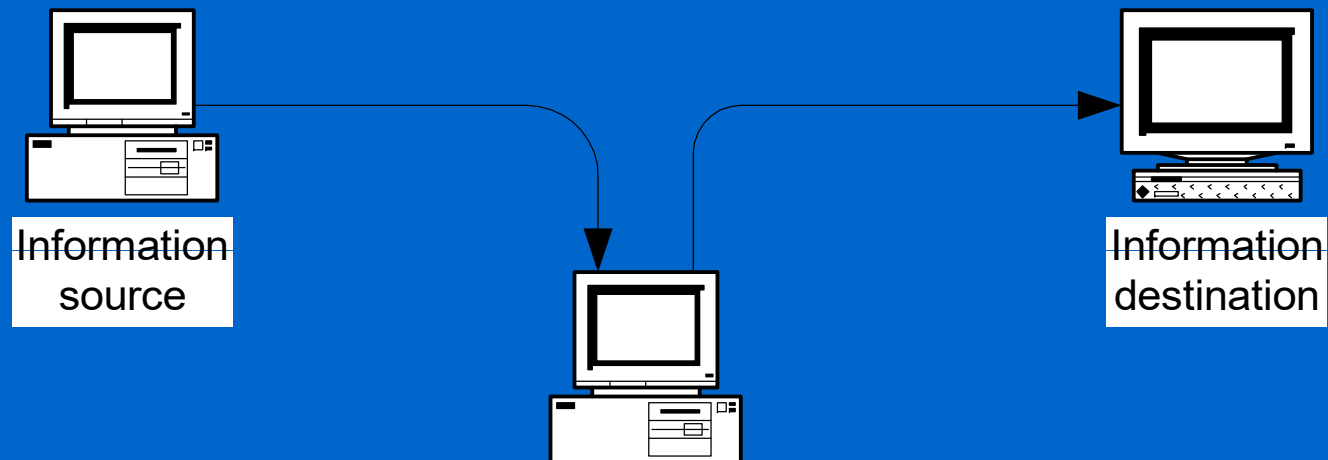
- An unauthorized party gains access to an asset
- This is an attack on **confidentiality**



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Modification

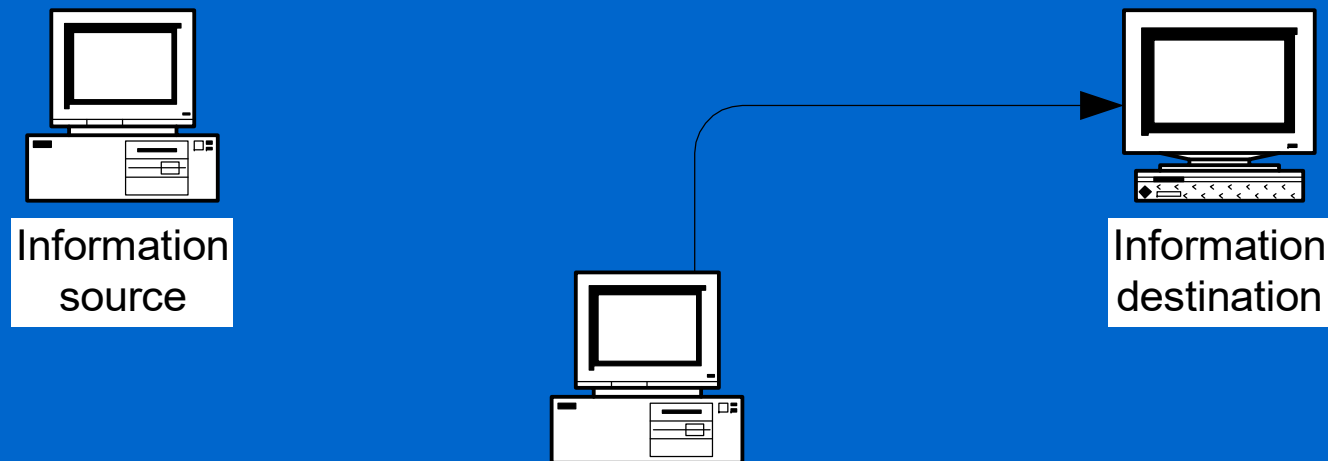
- An unauthorized party not only gains access to but tampers with an asset
- This is an attack on **integrity&authenticity**



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Fabrication

- An unauthorized party inserts counterfeit objects into the system
- This is an attack on **authenticity**



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Categories of Network Attacks

- **Passive vs. Active**

network security examples:

Passive threats

Reveal of
message contents
(Eavesdropping)

Traffic
analysis

Active threats

Masquerade
(spoofing,
hijacking)

Replay
(capture)

Modification
of message
contents
(tampering)

Denial of
service
(interrupti
on)

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Classes of S/W Security Vulnerabilities

- Buffer Overflow / Underflow, Integer Overflow
- Format Strings
- Tainted Input / Input Validation
- Race Conditions
- Trust Management
- Password Management
- Database Access (user ID/password)
- Insecure temp file usage, broken CGI practices
- Poor Cryptography Practices
- Poor Randomness

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Methods of Cryptoanalysis

focus on the **Encrypt/Decrypt** algorithm only

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- **Known Plaintext:** Eve has a bunch of ciphertexts and the corresponding plaintexts and tries to break a particular ciphertext.

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Ex: fixed letter head:

Dear Sir,

fixed file format:

<html>.....

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Methods of Cryptoanalysis(cont'd)

- **Chosen Plaintext:** Eve has a copy of ciphertext corresponding to a copy of plaintext selected by Eve who believes it is useful in breaking a ciphertext.

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Methods of Cryptoanalysis(cont'd)

- **Chosen Plaintext:** Eve has a copy of ciphertext corresponding to a copy of plaintext selected by Eve who believes it is useful in breaking a ciphertext. Eve can temporarily access the encryption engine

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- Ex: fighter plane transponder
challenge - response

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- **Chosen Ciphertext:** Eve has a copy of plaintext corresponding to a copy of ciphertext selected by Eve who believes it is useful in breaking a ciphertext. Eve can temporarily access the decryption engine.

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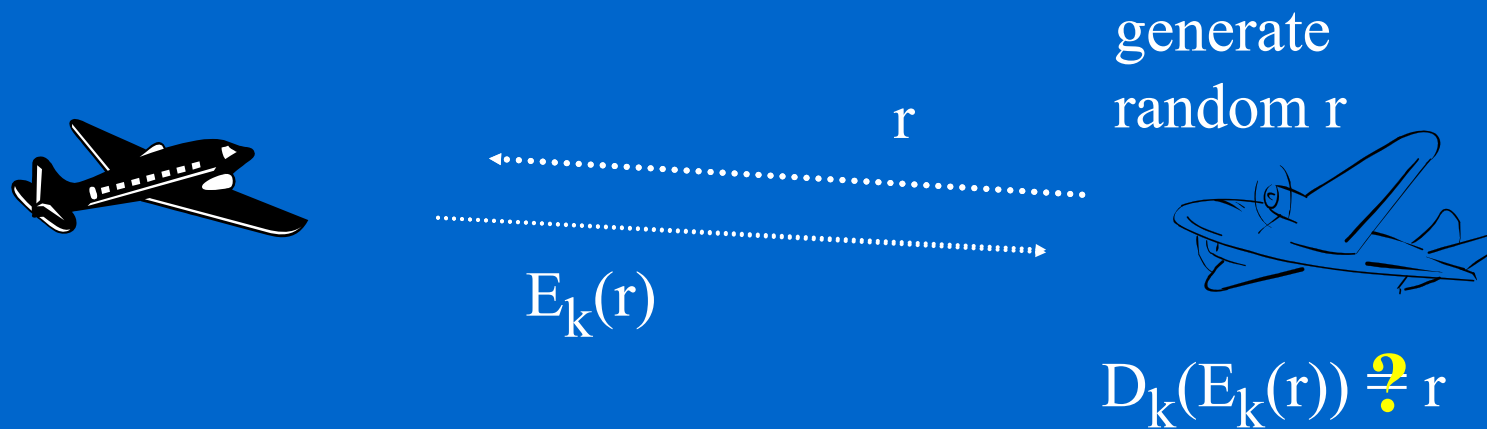
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- Ex: auto email response system

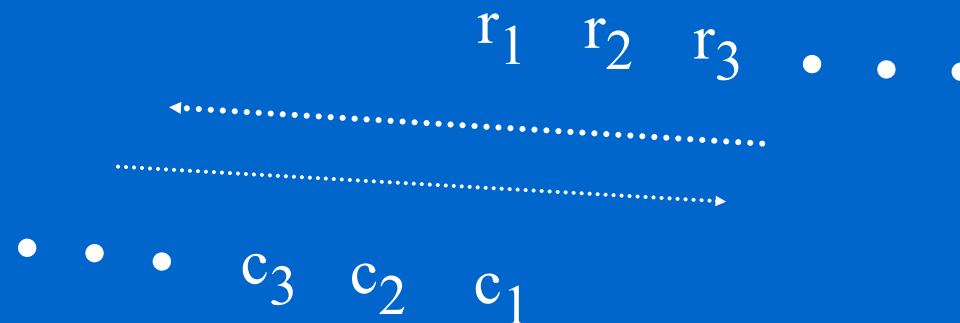
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Methods of Cryptoanalysis(cont'd)

- fighter plane transponder



- CPA:



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Symmetric & Public Key Algorithms

- **Symmetric Key Cryptosystems**
 - Encryption and decryption keys are known to both communicating parties (Alice and Bob).
 - They are usually related and it is **easy to derive from each other** (i.e. easy to derive the decryption key once one knows the encryption key and vice versa).
 - In most cases, they are identical.
 - All of the traditional (pre-1970) cryptosystems are symmetric.

Also known as secret-key cryptosystem

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Symmetric Key Cryptosystems

- Examples :
 - DES (Data Encryption Standard, 1976) and
 - AES (Advanced Encryption Standard, 2001): Rijndael
- A secret should be shared (or agreed) between communicating parties.

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Public Key Cryptography (PKC)

- **Why public key cryptography ?**
 - Key distribution and management are difficult in symmetric cryptosystems (DES, 3DES, IDEA, AES(Rijndael)) over large networks
 - Can not provide public verifiable and non-repudiable “digital signature” with symmetric ciphers
- Public key cryptography provides functions for all security services.
- Also makes it simple to implement key exchange, secret sharing functions, etc.

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Public Key Cryptosystems

- Each user has a pair of keys which are generated together under a scheme:
 - **Private Key** - known only to the owner
 - **Public Key** - known to anyone in the systems with validity assurance
- **Encryption with PKC:**
 - Sender encrypts the message by the **Public Key** of the receiver
 - Only the receiver can decrypt the message by her/his **Private Key**

asymmetry

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Non-mathematical PKC

the padlock metaphor

- Bob has a box and a padlock which only he can unlock once it is locked.
- Alice want to send a message to Bob.
- Bob sends his box and the unlocked padlock to Alice.
- Alice puts her message in the box and locks the box with Bob's padlock and sends the box to Bob thinking that the message is safe since only Bob can unlock the padlock and accesses the contents of the box.
- Bob receives the box, unlocks the padlock and reads the message.



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Non-mathematical PKC

- **Attack:**
 - Eve can replace Bob's padlock with hers when Bob is sending the box and padlock to Alice.

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Simple Puzzle

- 腐敗的俄羅斯郵政系統
 - 任何有價值,未上鎖的東西在經過郵政系統傳遞時安全抵達目的地的機會很接近 0
 - 聰明的俄羅斯人當然有辦法對付
 - **問題**: 有一個年輕人要送女友一枚貴重的戒指,他有一個很堅固的鐵盒,可以用鎖頭鎖住,請問他和他的女友該如何配合,才能運用郵政系統把戒指安全地寄達?? (假設鎖上的盒子能夠安全地寄達)

Shamir's three pass protocol

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Problems of PKC

- Powerful tools with their own intrinsic problems.
 - **Computationally intensive** operations are involved.
Much slower than the symmetric key algorithms.
PKC should not be used for encrypting large quantities of data.
 - Implementation is always a challenge.

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Example PKCs

- RSA
- Rabin
- Discrete Logarithm based cryptosystems.
(ElGamal)
- Elliptic Curve Cryptosystems
- Goldwasser-Micali
- Paillier
- Ajtai-Dwork
- Merkel-Hellman (Rivest-Chor)
- Cramer-Shoup

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Secret-Key vs Public-Key Systems

- Secret Key System offers
 - Information Secrecy (Privacy, Confidentiality)
 - Authentication (assuring that the other principal is the one who knows the shared key)
 - Integrity (using MAC)
- Disadvantages of a Secret Key System
 - key distribution/ key exchange
 - # of keys (key management)
 - can not offer non-repudiation

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Secret-Key vs Public-Key Systems (cont'd)

- Public Key System offers
 - information secrecy
 - key distribution / key management
 - non-repudiation
 - authentication and integrity
- Disadvantages of a Public Key System
 - **slow**, ex. RSA is 1000 times slower than DES (about 10^{-4} sec on a PIII 800 PC)
- Simple Designs
 - **cryptosystem** $D_{k_2}(E_{k_1}(m)) = m$
 - **signature system** $E_{k_1}(D_{k_2}(m)) = m$
- not every public key algorithm can be designed as both a cryptosystem and a signature system in this way, unless Encryption and Decryption algorithms are commutable

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Kerckhoffs's Principle (1883)

“Il faut qu’il n’exige pas le secret, et qu’il puisse sans inconvénient tomber entre les mains de l’ennemi.”

([A cipher] must not depend on secrecy, and it must not matter if it falls into enemy hands.)

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2. 電腦系統上駭客透過後門入侵很容易拿到整個程式

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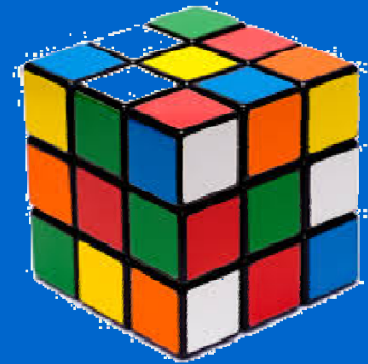
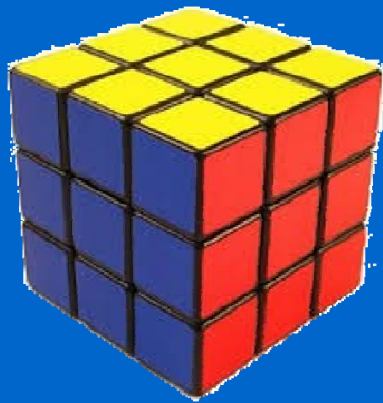
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Kerckhoffs's Principle 把整個加密過程 **完全公開** 的話，明文不就可以由密文推導出來了嗎???

關鍵在於「把加密系統拆開，
還保留一小部份祕密」
加密鑰匙 (祕密) + 加密演算法 (公開)

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Decomposition of an algorithm



12 possible moves

Conceptually:

- ① choices of moves
- ② difficult to resolve the choices reversely
- ③ difficult to solve in a brute-force way



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Strength of a Cryptosystem

- While assessing the strength of a cryptosystem, one should always assume that **the enemy knows the cryptographic algorithm used.** (Kerkckhoff's Principle)

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- The security of an encryption system should be based on
 - the **quality of the algorithm** but not its obscurity
 - the key space (or **key length**)

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 - the key space (or **key length**)
- The quality of cryptographic algorithms is measured through cryptanalysis (usually complicated and difficult)

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Key Length in Cryptosystems

- However, the key space should be large enough to prevent the adversary to determine the key simply by trying all possible keys in the key space.
- This is called **brute force** or **exhaustive search** attack.
- For example, DES utilizes 56-bit key, therefore there are 2^{56} (or approx 7.2×10^{16}) possible keys in the key space

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Key Length in Cryptosystems

- Assume that there are 10^{30} possible keys to be tried and you can only try 10^9 keys in a second.
- Since there are only around 3×10^7 seconds in a year, brute force attack would take more than 3×10^{13} years to try all the keys over. This duration is longer than the predicted life of the universe.

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Key Length in Cryptosystems

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- S/He needs to take advantage the weakness in the algorithm or in the implementation of the cipher in order to reduce the possible keys to try out.
- However, in some cases, longer keys do not necessarily improve the security

NSA Suite-B: Security Strength of Practical Algorithms

Security Strength (bits)	Symmetric Key	Asymmetric Key	Elliptic Curve Asymmetric Key	Message Digest
80 (too weak by 2010)	Triple DES (2 key)	1024-bit RSA / DSA	160-bit ECDSA	SHA-1
112 (too weak by 2030)	Triple DES (3 key)	2048-bit RSA / DSA	224-bit ECDSA	SHA-224
128	128-bit AES	3072-bit RSA / DSA	256-bit ECDSA	SHA-256
192	192-bit AES	7680-bit RSA / DSA	384-bit ECDSA	SHA-384
256	256-bit AES	15360-bit RSA / DSA	512-bit ECDSA	SHA-512

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Large Numbers

Physical Analogue	Number
Odds of being killed by lightning (per day)	1 in 9 billion (2^{33})
Odds of winning the top prize in a US state lottery	1 in 4,000,000 (2^{22})
Odds of winning the top prize in a US state lottery and being killed by lightning in the same day	1 in 2^{55}
Odds of drowning in the US per year	1 in 59,000 (2^{16})
Odds of being killed in an automobile accident in the US (in 1993)	1 in 6100 (2^{13})
Odds of being killed in an automobile accident in the US per lifetime	1 in 88 (2^7)
Time until next ice age	14,000 (2^{14}) years
Time until the sun goes nova	10^9 (2^{30}) years
Age of the planet	10^9 (2^{30}) years
Age of the universe	10^{10} (2^{34}) years
Number of atoms in the planet	10^{51} (2^{170})
Number of atoms in the sun	10^{57} (2^{190})
Number of atoms in the galaxy	10^{61} (2^{223})
Number of atoms in the universe (dark matter excluded)	10^{77} (2^{265})
Volume of the universe	10^{84} (2^{280}) cm^3

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Large Numbers

If the Universe is Closed:

Total lifetime of the Universe

$10^{11}(2^{37})$ years

$10^{18}(2^{61})$ seconds

If the Universe is Open:

Time until low-mass stars cool off

$10^{14}(2^{47})$ years

Time until planets detach from stars

$10^{15}(2^{50})$ years

Time until stars detach from galaxies

$10^{19}(2^{64})$ years

Time until orbits decay by gravitational radiation

$10^{20}(2^{67})$ years

Time until black holes decay by the Hawking process

$10^{64}(2^{213})$ years

Time until all matter is liquid at zero temperature

$10^{65}(2^{216})$ years

Time until all matter decays to iron

$10^{10^{26}}$ years

Time until all matter decays to black hole

$10^{10^{76}}$ years

The above data comes from Schneier's "Applied Cryptography," 1996

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Key Length in Cryptosystems

**Symmetric and Public-key Key Lengths
with Similar Resistances to Brute-Force Attacks**

Symmetric Key Length	Public-key Key Length
56 bits	384 bits
64 bits	512 bits
80 bits	768 bits
112 bits	1792 bits
128 bits	2304 bits

The above data comes from Schneier's "Applied Cryptography," 1996

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Chinese Number Systems

- 中國古代的【孫子算經】一書中有記載：
「凡大數之法，萬萬曰億，萬萬億曰兆，萬萬兆曰京，萬萬京曰垓（讀做 ㄍㄞˋ），萬萬垓曰秭（讀做 ㄗˇ），萬萬秭曰穰（讀做 ㄖㄨㄥˊ），萬萬穰曰溝，萬萬溝曰澗，萬萬澗曰正，萬萬正曰載。」
- 隨著印度佛經的傳入中國，而增加了恆河沙、阿僧祇、那由他、不可思議、無量等，這些數詞都出現在佛經中，用來計量時間的長度

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Chinese Number System (cont'd)

2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	9	8	7	6	5	4	3	2	1	0
4	3	2	1	0	9	8	7	6	5	4	3	2	1	0										
秭	千垓	百垓	十垓	垓	千京	百京	十京	京	千兆	百兆	十兆	兆	千億	百億	十億	億	千萬	百萬	十萬	萬	千	百	十	個

10^x



72	68	64	60	56	52	48	44	40	36	32	28	24	20	16	12	8	4	3	2	1	0
大數	無量	不可思議	那由他	阿僧祇	恆河沙	極	載	正	澗	溝	穰	秭	垓	京	兆	億	萬	千	百	十	個

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References

- Cryptography: Theory and Practice, 3rd Ed., D. Stinson
- Cryptography: An Introduction, N. Smart

