



# Classical Ciphers



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# Classical Cryptography

- Monoalphabetic ciphers: letters of the plaintext alphabet are mapped into unique ciphertext letters
- Polyalphabetic ciphers: letters of the plaintext alphabet are mapped into ciphertext letters depending on the context of the plaintext
- Stream ciphers: a key stream is generated and used to encrypt the plaintext

# Classical Cryptosystem: Shift Cipher

- **Shift Cipher**

- Letters of the alphabet are assigned unique numbers

a	b	c	d	e	f	g	h	i	j	k	l	m
0	1	2	3	4	5	6	7	8	9	10	11	12
n	o	p	q	r	s	t	u	v	w	x	y	z
13	14	15	16	17	18	19	20	21	22	23	24	25

- **Algorithm:**

- Let  $P = C = K = Z_{26}$  and  $x \in P, Y \in C, k \in K$
- *Encryption:*  $E_k(x) = x + k \bmod 26.$
- *Decryption:*  $D_k(Y) = Y - k \bmod 26.$

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# Shift Cipher

- *Caesar Cipher* : shift cipher with  $k = 3$
- **Example:** Let the key  $k = 17$ 
  - Plaintext:  $X = a t t a c k = (0, 19, 19, 0, 2, 10)$
  - Ciphertext :  $Y = (0+17 \bmod 26, 19+17 \bmod 26, \dots)$   
 $= (17, 10, 10, 17, 19, 1) = R K K R T B$
- **Attacks**
  - Ciphertext only:
    - **Exhaustive Search:** Try all possible keys.  $|K|=26$ .  
Nowadays, for moderate security  $|K| \geq 2^{80}$ ,  
for recommended security  $|K| \geq 2^{100}$ .
    - **Letter frequency analysis** (Same plaintext maps to same ciphertext)

# Frequency Analysis

- In most languages, letters occur in texts with different frequencies
- single, double, triple letter frequencies

Single	Frequency	Double	Triple
E	.127	TH	THE
T	.091	HE	ING
A	.082	IN	AND
O	.075	ER	HER
I	.070	AN	ERE
N	.067	RE	ENT
S	.063	ED	THA
H	.061	ON	NTH

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# Letter Frequency Analysis

- **Method 1:** Find the most frequent cipher character, make a guess as  $E_k('e')$ , solves  $k$ . Use this  $k$  to decrypt ciphertext and see if it is a reasonable guess. Otherwise, find the second frequent cipher character, make a guess as  $E_k('e')$ .

- **Method 2:** correlation

$$A_0 = \begin{bmatrix} .082 & .015 & .028 & .043 & .127 & .022 & .020 & .061 & .070 & .002 \\ .008 & .040 & .024 & .067 & .075 & .019 & .001 & .060 & .063 & .091 \\ .028 & .010 & .023 & .001 & .020 & .001 & & & & \end{bmatrix}$$

$A_i$  is obtained by circularly shift right  $A_0$   $i$  elements

e.g.  $A_2 = [.020 \ .001 \ .082 \ .015 \ .028 \ .043 \ \dots]$

- correlation =  $A_i \cdot A_j$  is the usual dot product between  $A_i$  and  $A_j$
- let  $A$  be the frequency of the ciphertext paragraph
- calculate correlation between  $A$  and  $A_i$ , choose the maximum

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# Shift Cipher

- **Known plaintext:** You can deduce the key if you know one letter of the plaintext along with its corresponding ciphertext. Ex. t(=19) encrypts to D(=3), then the key is  
$$k \equiv 3 - 19 \equiv -16 \equiv 10 \pmod{26}$$
- **Chosen plaintext:** choose the letter 'a' as the plaintext, the ciphertext is the key
- **Chosen ciphertext:** choose the letter 'A' as ciphertext, the plaintext is the negative of the key

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## Shift Cipher

- One time pad can be considered as a shift cipher with **modulus 2** and a changing key sequence, in which each key is used only for one plaintext character and never repeated.
- A shift cipher as defined is therefore perfectly secure if the key keeps changing and is used for one character only.

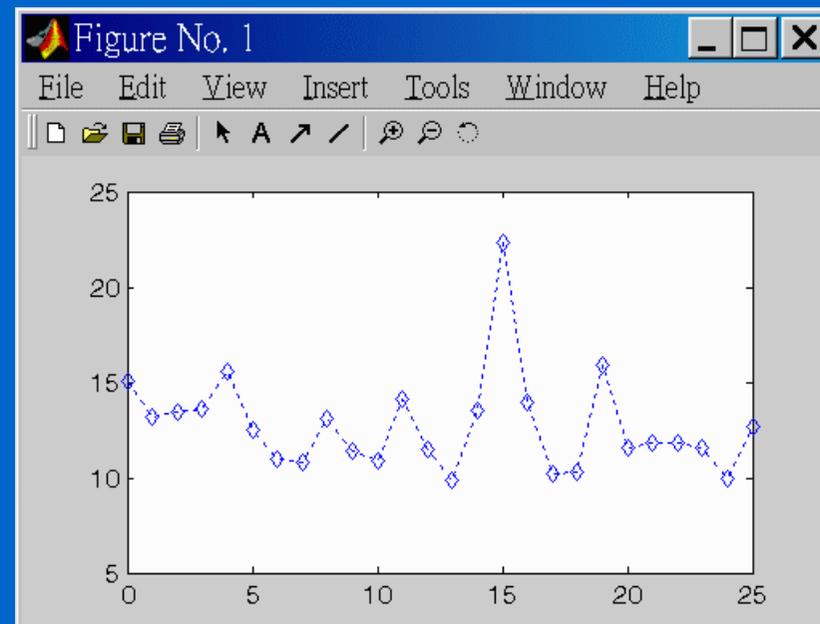
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## Matlab Example

- `dir, cd, help`
- `path(path, 'c:\lcwMatlabCode')`
- `k = 20`  
`plain = 'hellothisisashiftcipherexample'`  
`plain_i = text2int(plain)`  
`cipher_i = mod(plain_i + k, 26)`  
`cipher = int2text(cipher_i)`  
`recovered_i = mod(cipher_i - k, 26)`  
`recovered = int2text(recovered_i)`
- `cipher = shift(plain, k)`  
`recovered = shift(cipher, -k)`

# Matlab letter frequency analysis

- sci=  
['themethodusedforthepreparationandreadingofcodemessagesissimplei', ...  
'ntheextremeandatthesametimeimpossibleoftranslationunlesssthekeyi', ...  
'sknowntheeasewithwhichthekeymaybechangedisanotherpointinfavorof', ...  
'theadoptionofthiscodebythosedesiringtotransmitimportantmessages', ...  
'withouttheslightestdangeroftheirmessagesbeingreadbypoliticalorb', ...  
'usinessrivalsetc'];
- cipher=shift(sci, 15);
- freq=frequency(cipher);
- correlation=corr(freq);
- plot(0:25,correlation,'bd:')



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## Affine Cipher

- **Algorithm:** Let  $P = C = \mathbb{Z}_{26}$  and  $x \in P, Y \in C$ 
  - *Encryption:*  $E_k(x) = Y = \alpha \cdot x + \beta \pmod{26}$
  - The key  $k = (\alpha, \beta)$  and  $\alpha, \beta \in \mathbb{Z}_{26}$
  - ex.  $\alpha=13, \beta=4$ 
    - input = (8, 13, 15, 20, 19)  $\Rightarrow$  (4, 17, 17, 4, 17) = ERRER
    - alter = (0, 11, 19, 4, 17)  $\Rightarrow$  (4, 17, 17, 4, 17) = ERRER
  - There is no one-to-one mapping between plaintext and ciphertext. What's wrong?
  - *Decryption:*  $D_k(Y) = x = \alpha^{-1} \cdot (Y - \beta)$   
 $= \alpha' \cdot Y + \beta' \pmod{26}$

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# Affine Cipher

- **Key Space:**
  - $\beta$  can be any number in  $Z_{26}$ . 26 possibilities
  - Since  $\alpha^{-1}$  is required to exist, we can only select integers in  $Z_{26}$  s.t.  $\gcd(\alpha, 26) = 1$ . Candidates are  $\{1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25\}$
  - Therefore, the key space has  $12 \cdot 26 = 312$  candidates.
- **Attack types:**
  - *Ciphertext only*: exhaustive search or frequency analysis

# Letter Frequency Analysis

- Consider the ciphertext

FMNVEDKAPHFERBNDKRX  
RSREFMORUDSDKDVSHVU  
FEDKAPRKDLYEVLRRHRH

- Letter frequency of the ciphertext:

Letter	# of Occurrences
R	8
D	6
E	5
H	5
K	5
V	4
F	4

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# Letter Frequency Analysis

- Make a guess: choose two potential candidate letters  
e.g. 1st guess R  $\rightarrow$  e and D  $\rightarrow$  t
- Try to show the guess make sense by solving  
 $(\alpha, \beta)$  from  $E_k(x) = Y = \alpha \cdot x + \beta \pmod{26}$   
e.g.  $4\alpha + \beta = 17 \pmod{26}$  and  $19\alpha + \beta = 5 \pmod{26}$   
 $\Rightarrow \alpha = 6, \beta = 19$ , which is illegal since  $\gcd(6, 26) > 1$
- 2nd guess: R  $\rightarrow$  e and E  $\rightarrow$  t .....  $\Rightarrow \alpha = 13$ , still illegal
- 3rd guess: R  $\rightarrow$  e and H  $\rightarrow$  t .....  $\Rightarrow \alpha = 3, \beta = 5$   
i.e.  $E_k(x) = 3 \cdot x + 5 \pmod{26}$   
 $D_k(x) = 9 \cdot x - 19 \pmod{26}$

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# Letter Frequency Analysis

- Better Solution: correlation
  - Enumerate 312 possible keys, ex. (3,2)
  - Let  $A_0 = [.082, .015, .028, .043, .127, .022, .020, .061, .070, .002, .008, .040, .024, .067, .075, .019, .001, .060, .063, .091, .028, .010, .023, .001, .020, .001]$
  - Let the  $i$ -th key be (3,2), which maps plaintexts [0, 1, 2, 3, 4 ..., 25] to ciphertexts [2, 5, 8, 11, 14, 17, 20, 23, ...]
  - Calculate a vector  $A_i$  with the  $k$ -th element being  $A_0(E_{3,2}(k))$ , ex.  $A_i = [A_0(2), A_0(5), A_0(8), A_0(11), A_0(14), A_0(17), A_0(20), A_0(23), A_0(0), \dots]$
  - Perform correlation  $A \cdot A_i$  and find the maximum

# Affine Cipher

- **Attack types:**

- *Known plaintext*: **two letters** in the plaintext and corresponding ciphertext letters would suffice to find the key.

Ex. plaintext 'if'=(8, 5) and ciphertext 'PQ'=(15, 16)

$$8 \cdot \alpha + \beta \equiv 15 \pmod{26}$$

$$5 \cdot \alpha + \beta \equiv 16 \pmod{26} \Rightarrow \alpha = 17 \text{ and } \beta = 9$$

What happens if we have only one letter of known plaintext?

still have great reduction in candidates

# Affine Cipher

- **Attack types:**
  - *Chosen plaintext*: Choose a and b as the plaintext. The first character of the ciphertext will be equal to  $0 \cdot \alpha + \beta = \beta$  and the second will be  $\alpha + \beta$ .
  - *Chosen ciphertext*: Choose A and B as the ciphertext. The first character of the plaintext will be equal to  $0 \cdot \alpha' + \beta' = \beta'$  and the second will be  $\alpha' + \beta'$ ,  $\alpha = (\alpha')^{-1}$  and  $\beta = -\alpha \cdot \beta'$

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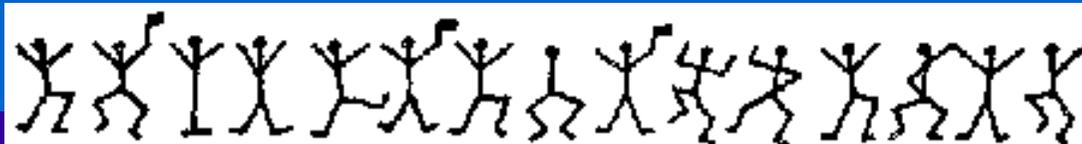
## Matlab Example

- $a = 3, b = 5, ap = 9, bp = -19;$
- `plain = 'matlabaffinecipherencryptionexample';`
- `cipher = affinecrypt(plain, a, b)`
- `recovered = affinecrypt(cipher, ap, bp)`

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# Substitution Ciphers

- Each letter in the alphabet is replaced (substituted) by another letter. More precisely, a permutation of the alphabet is chosen and applied to the plaintext.
- Shift ciphers and affine ciphers are special cases of substitution ciphers.
- Since ciphertext preserves the statistic of the language used in the plaintext, the “frequency analysis” is an effective way of breaking substitution ciphers with only ciphertext.
- The Adventure of the Dancing Men by Arthur Conan Doyle <http://www.sherlockian.net/canon/stories/danc.html>





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## Vigenère Cipher

- Finding the key length:
  - Friedman's Test uses **Index of Coincidence**: Let  $I_c(x)$  be the probability that two random elements of the n-letter string  $x$  are identical
  - Let  $f_0, f_1, \dots, f_{25}$  be the number of occurrence of A, B, ...Z, respectively in the n-letter string  $x$

$$I_c(x) = \frac{\sum_{i=0}^{25} \binom{f_i}{2}}{\binom{n}{2}} = \frac{\sum_{i=0}^{25} f_i(f_i - 1)}{n(n - 1)}$$

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## Vigenère Cipher

- The letter frequency of English is
  - $A_0 = [.082 \ .015 \ .028 \ .043 \ .127 \ .022 \ .020 \ .061 \ .070 \ .002$   
 $\ .008 \ .040 \ .024 \ .067 \ .075 \ .019 \ .001 \ .060 \ .063 \ .091$   
 $\ .028 \ .010 \ .023 \ .001 \ .020 \ .001]$
- The expected value of  $I_c(x)$  is
  - for **English Text**:  

$$I_c(x) = A_0 \cdot A_0 = (.082)^2 + (.015)^2 + \dots = 0.666$$
  - for **Random String**:  

$$I_c(x) = 26 \cdot (1/26)^2 = 0.038$$
  - for shifted English Text (the first letter shifted by  $k_i$  and the second letter shifted by  $k_j$ ):

$ i - j $	1	2	3	4	5	6	7	8	9	10	11	12	13
$I_c(x) = A_i \cdot A_j$	.039	.032	.034	.044	.033	.036	.039	.034	.034	.038	.045	.039	.042

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# Vigenère Cipher

- find the coincidences in the ciphertext

'vvhqwvvrhmusgigthkihtssejchlsfcbgvwerlryqtfsvgahwkcuhauglq'  
 'hnsrljshbltspisprdxljsveeghlqwkasskuwepwqtwwspgoelkcqyfnsv'  
 'wljsniqkgnrgybwlwgoviokhkazkqkxzgyhcecmuijoqkwfwvefqhkijrc'  
 'lrlkbienqfrljdsdgrhlsfqtwlauqrhwdmwlwgusgikkflryvcwvspgpmlk'  
 'assjvoqxeggveyggzmljcxljsvpaivwikvrdrygfrljslveggveyggeia'  
 'puisfpbtgnwwmuczrvtwglrwugumnczville'

	shift				
	→				
• coincidences:	14	14	16	14	24
	12	13	13	7	14
(by shift and count)	13	19	13	15	26
	11	13	14	11	20
	17	14	15	16	21

⇒ Key length is 5

# Vigenère Cipher

- Finding the Key:

- To find the first element of the key, count the frequencies of the letters in the 1st, 6th, 11th ... positions of the ciphertext

$$V = (0,0,7,1,1,2,9,0,1,8,8,0,0,3,0,4,5,2,0,3,6,5,1,0,1,0)$$

- Divide by number of letters counted, 67

$$W = (0, 0, .1045, .0149, .0149, .0299, \dots, .0149, 0)$$

- Compute  $W \cdot A_i$  for  $0 \leq i \leq 25$

0.0250	0.0391	0.0713	0.0388	0.0275	0.0380	0.0512	0.0301	0.0325
0.0430	0.0338	0.0299	0.0343	0.0446	0.0356	0.0402	0.0434	0.0502
0.0392	0.0296	0.0326	0.0392	0.0366	0.0316	0.0488	0.0349	

⇒ first key is 'c'

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## Vigenère Cipher

– Known plaintext:

- if enough (plaintext, ciphertext) pairs are known

$$k_i = Y - x$$

– Chosen plaintext:

- choose plaintext aaaaa...

$$k_i = Y$$

– Chosen ciphertext:

- choose ciphertext AAAAA...

$$k_i = -x$$

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# Matlab Example

- Encrypt/decrypt
  - `key = 'vigenere';`
  - `key_i = text2int(key);`
  - `plain = 'matlabaffinecipherencryptionexample';`
  - `cipher=vigenere(plain, key_i)`
  - `recovered=vigenere(cipher, -key_i)`

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# Matlab Example

- Ciphertext only attack:

- ciphertexts

- for i=1:25,

- a(i) = coinc(vvhq, i);

finding key length

- end

- first = choose(vvhq, 5, 1)

- V = frequency(first)

finding first key

- W = V / length(first)

- corr(W)

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# Block Ciphers

- In the substitution ciphers, changing one letter in the plaintext changes exactly one letter in the ciphertext.
- This greatly facilitates finding the key using frequency analysis.
- Block ciphers prevent this by encrypting a block of letters simultaneously.
- Many of the modern (symmetric) cryptosystems are block ciphers. DES operates on 64 bits of blocks while AES uses 128 bits of blocks (optionally 192 and 256 bits blocks).

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# Hill Cipher

- The key is an  $n \times n$  matrix whose entries are elements in  $\mathbb{Z}_{26}$
- Ex. Let  $n=3$ , the key matrix be

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 11 & 9 & 8 \end{pmatrix}$$

and the plaintext be **abc** = (0, 1, 2) then the encryption operation is a vector-matrix multiplication

$$(0,1,2) \times \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 11 & 9 & 8 \end{pmatrix} \equiv (0,23,22) \pmod{26} \Rightarrow \text{AXW (ciphertext)}$$

In order to decrypt, the inverse of the key matrix M is:

$$N = \begin{pmatrix} 22 & 5 & 1 \\ 6 & 17 & 24 \\ 15 & 13 & 1 \end{pmatrix}$$

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## Hill Cipher (cont'd)

- If we change one letter in the plaintext, all the letters of the ciphertext will be affected.
- Let the plaintext be bbc instead of abc then the ciphertext

$$(1,1,2) \times \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 11 & 9 & 8 \end{pmatrix} \equiv (1,25,25) \pmod{26} \Rightarrow \text{BZZ (ciphertext)}$$



# General Design Principle

- Claude Shannon, in *Communication theory of secrecy systems* Bell Systems Technical Journal 28, (1949), 656-715, introduced properties that a good cryptosystems should have:
  - **Diffusion**: one character changes in the plaintext should effect as many ciphertext characters as possible, and vice versa.
  - **Confusion**: The key should not relate to the ciphertext in a simple way.

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# Stream Cipher

- plaintext alphabets  $P$   
ciphertext alphabets  $C$   
key stream alphabet  $L$   
key stream generator  $F = \{f_1, f_2, \dots\}$   
 $f_i: K \times P^{i-1} \rightarrow L$   
 $\ell_i = f_i(k, x_1, \dots, x_{i-1})$        $k$  is the seed
- Encryption:
  - for plaintext  $x_1, x_2, \dots$  ciphertext  $c_1 = E_{\ell_1}(x_1), c_2 = E_{\ell_2}(x_2), \dots$
- Decryption:
  - for ciphertext  $c_1, c_2, \dots$  recovered plaintext  $x_1 = D_{\ell_1}(c_1), x_2 = D_{\ell_2}(c_2), \dots$
- For each  $\ell \in L$ ,  $E_\ell, D_\ell$  satisfy  $\forall x \in P, D_\ell(E_\ell(x)) = x$

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# Autokey cipher

- Key stream generator:  $\ell_i = x_{i-1}$ ,  $\ell_1 = k$ ,  $k$  is an initial seed  
 Encryption:  $E_\ell(x) = x + \ell \pmod{26}$   
 Decryption:  $D_\ell(y) = y - \ell \pmod{26}$
- Ex:  $k = 8$ , plaintext: 'rendezvous'

plaintext:	{	r	e	n	d	e	z	v	o	u	s	
		17	4	13	3	4	25	21	14	20	18	
keys:		8	17	4	13	3	4	25	21	14	20	18
ciphertext:	{	25	21	17	16	7	3	20	22	8	12	
		Z	V	R	Q	H	D	U	J	I	M	
keys:		8	17	4	13	3	4	25	21	14	20	18
plaintext:		17	4	13	3	4	25	21	14	20	18	

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# Stream Cipher

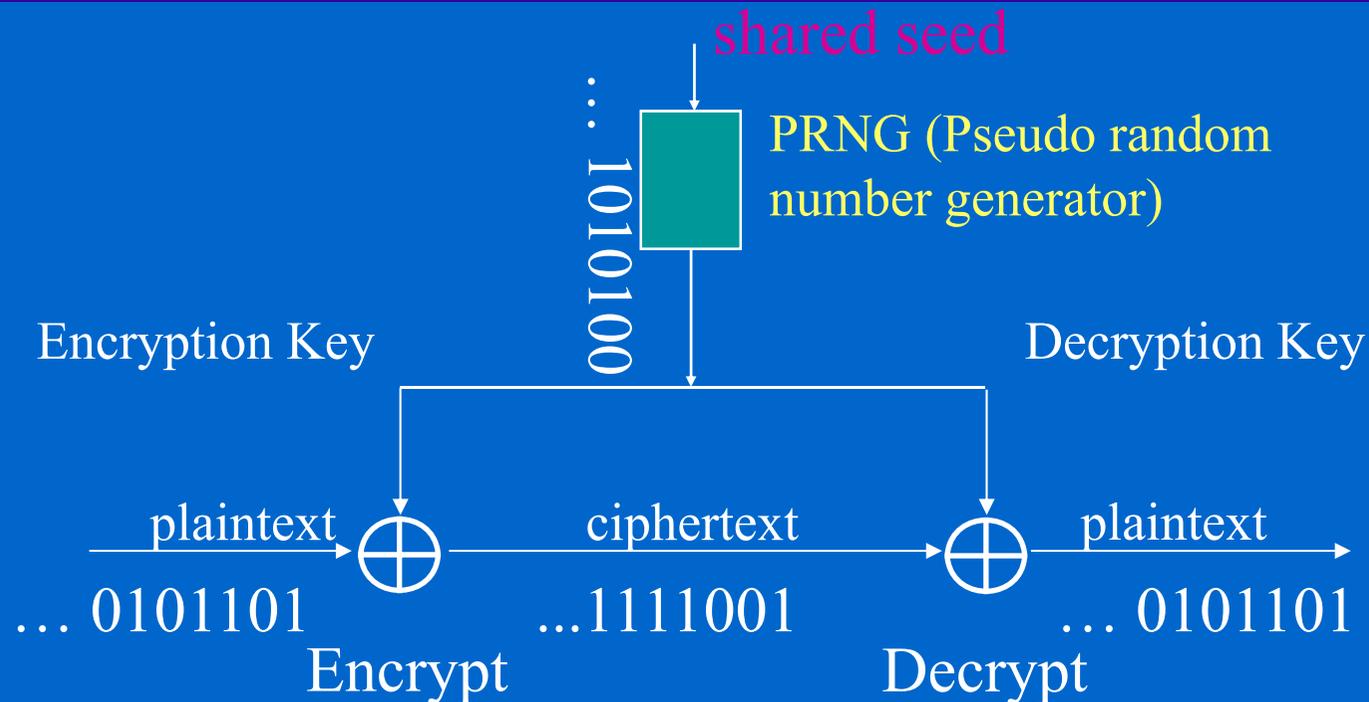
- **Block ciphers** are special cases of stream ciphers where the key stream is constant.
- A stream cipher is **synchronous** if the key stream is independent of the plaintext.
  - Both sender and receiver must be synchronized.
  - Resynchronization can be needed.
  - No error propagation (if the deciphered plaintext is incorrect).
  - Active attacks can easily be detected.
- A stream cipher is **periodic** with period  $d$  if  $\ell_{i+d} = \ell_i$ , for all  $i \geq 1$ .

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## Stream Cipher

- The Vigenère cipher with keyword length  $m$  is a **periodic** stream cipher with period  $m$ .
- Stream ciphers are often described in binary 0, 1 alphabets. ex. one-time pad
- Perfectly Secure: One-time pad
- Examples of practical stream ciphers
  - Autokey Cipher
  - One-time pad with Pseudo-random Bit Generation
  - Linear Feedback Shift Register (LFSR)
  - DES in Counter Mode or CFB Mode
  - Feistel Cipher

# OTP with PRNG



- Security? not provably secure??
  - Random number sequence is used as key!! What if it is not so random??
- What is a random number?

# Randomness

- Randomness? ex. flipping a fair coin, thermal noise

random

- **Uniformly distributed string sequences**
- a string  $s$  is Komogorov-random if its length equals the length of the shortest program producing  $s$   
ex. 010101010101010101

pseudo random, PRNG

- **Statistical approach:** pass some statistical tests: ex. 0/1 bits appear equally, number of 0/1 bits are equal, any two bits are uncorrelated, Maurer's Universal Test, Chi-Square Test, Kolmogorov-Smirnov Test ...
- **Computational approach:**
  - indistinguishable from any uniformly distributed sequences
  - unpredictable by any poly-time algorithm (the probability to predict the next bit is no better than  $1/2$ )

# Pseudorandom Number Generator

- Existence? one way function assumption
- Poor implementation for cryptographic usage:
  - linear congruential generator rand() in the standard C/UNIX library
    - $x_n = a x_{n-1} + b \pmod m$ ,  $x_0$  is the initial seed
    - $a, b, m$  can be discovered from the  $x_n$  sequence
    - therefore  $x_n$  is completely predictable (key is know to everybody!!)
    - any polynomial congruential generator is cryptographically insecure
    - can be used only for the purpose of statistical experiments

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# Pseudorandom Number Generator

- Fairly good implementation for cryptographic purpose:
  - Method 1: based on one-way function candidates (DES, SHA..)
    - one-way function  $f$ :  $y = f(x)$ , given  $y$ , it's hard to compute  $x$   
 $x_j = f(s+j)$ ,  $j=1,2,3,\dots$   $s$  is the seed  
let the random bit sequence  $b_j$  be the LSB of  $x_j$ ,
    - PRNG in the OpenSSL toolkit is based on SHA
  - Method 2: Blum-Blum-Shub (BBS, 1984)
    - $p \equiv 3 \pmod{4}$ ,  $q \equiv 3 \pmod{4}$ ,  $n = p \cdot q$ , seed  $k$
    - $x_0 \equiv k^2 \pmod{n}$ ,  $x_j \equiv x_{j-1}^2 \pmod{n}$ ,  
let the random bit sequence  $b_j$  be the LSB of  $x_j$

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## BBS example

- Let  $p = 24672462467892469787$   $q = 396736894567834589803$   
 $n = 9788476140853110794168855217413715781961$   
take  $k = 873245647888478349013$   
 $x_0 \equiv k^2 \pmod{n} \equiv 8845298710478780097089917746010122863172$   
 $x_1 \equiv x_0^2 \pmod{n} \equiv 7118894281131329522745962455498123822408$   
 $x_2 \equiv x_1^2 \pmod{n} \equiv 3145174608888893164151380152060704518227$   
....  
 $b_1 = 0$   
 $b_2 = 1, \dots$
- slow for practical application,  
take  $k (\leq \log_2 \log_2 n)$  LSB bits of  $x_j$

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## Maple example in Matlab

```
maple('p := 24672462467892469787')
```

```
maple('q := 396736894567834589803')
```

```
maple('n := p*q')
```

```
maple('x := 873245647888478349013')
```

```
maple('x0 := x&^2 mod n')
```

```
maple('x1 := x0&^2 mod n')
```

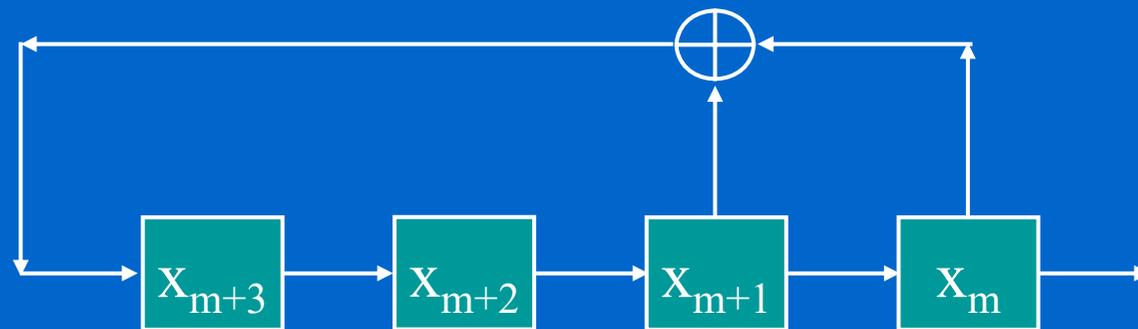
```
maple('x2 := x1&^2 mod n')
```

...

```
mhelp intro  
mhelp mod  
mhelp ^  
mhelp &  
mhelp :=
```

# Linear Feedback Shift Register (LFSR)

- Hardware-oriented implementation: sacrifice security to obtain encryption speed



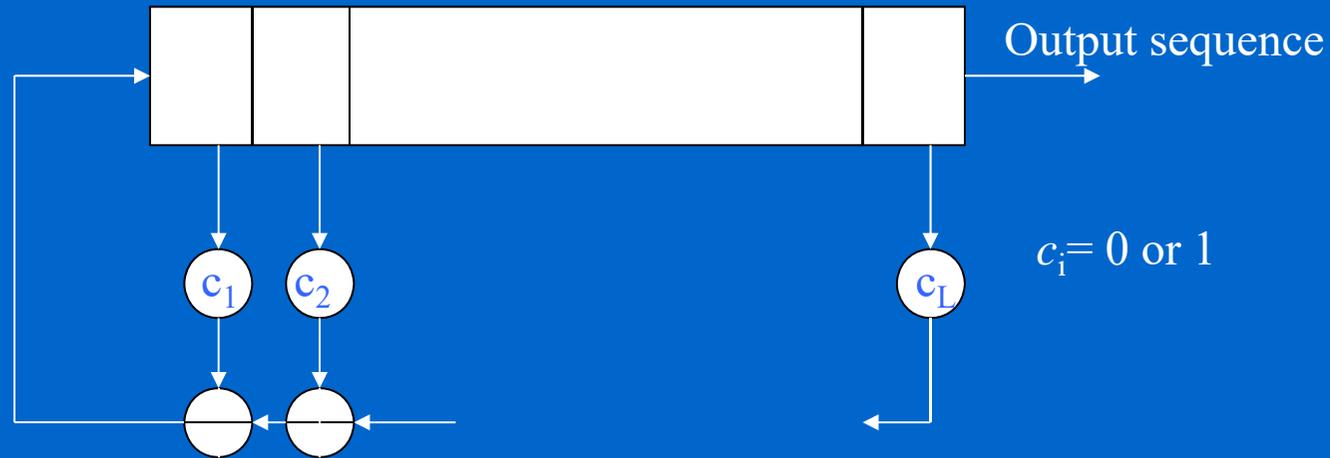
$$X_{m+3} = X_{m+1} + X_m$$

- in general:

$$X_{n+m} = c_0 X_n + c_1 X_{n+1} + \dots + c_{m-1} X_{n+m-1} \pmod{2}$$

with initial values  $x_1, x_2, \dots, x_m$

# Linear Feedback Shift Register (LFSR)



$$C(x) = c_1x + c_2x^2 + \dots + c_Lx^L$$

- If  $C(x)$  is primitive, LFSR is called *maximum-length LFSR*, and the output sequence is called *m-sequence* and its period is  $T = 2^L - 1$ .
- *m-sequences* have good statistical properties.
- However, they are predictable.

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# Linear Feedback Shift Register (LFSR)

- For a length  $m$  linear recurrence relation, the period of the sequence is at most  $2^m - 1$ .
  - Any  $m$  consecutive terms of the sequence determine the complete sequence. As soon as there are **more than**  $2^m - 1$  terms, some string of length  $m$  must occur twice.



- ex.  $x_{n+31} \equiv x_n + x_{n+3}$ , with any **nonzero initial vector**, will produce a sequence that has period  $2^{31} - 1$

# Linear Feedback Shift Register (LFSR)

- Given a segment 011010111100 of a LFSR sequence, it is possible to deduce the length of the recurrence and the coefficients. (If you find a segment of  $2m$ -bit plaintext and the corresponding ciphertext, you discover the corresponding segment of the key sequence.)
- The general solution: solve coefficients  $c_i$  from

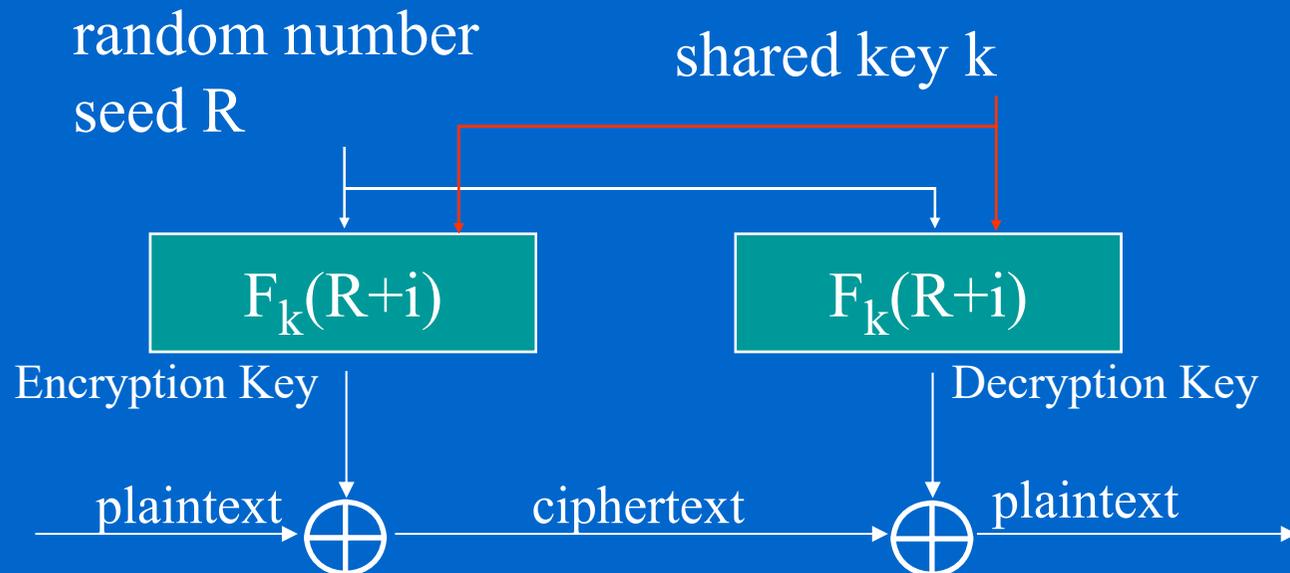
$$\begin{pmatrix} x_1 & x_2 & \cdots & x_m \\ x_2 & x_3 & \cdots & x_{m+1} \\ \vdots & \vdots & \vdots & \vdots \\ x_m & x_{m+1} & \cdots & x_{2m-1} \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{m-1} \end{pmatrix} \equiv \begin{pmatrix} x_{m+1} \\ x_{m+2} \\ \vdots \\ x_{2m} \end{pmatrix}$$

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## Linear Feedback Shift Register (LFSR)

- Computation in  $GF(2^n)$  can be quickly implemented in hardware with linear-feedback shift registers.
- Computation in  $GF(2^n)$  (eg. exponentiation and discrete log) is often quicker than computation over  $GF(p)$ .
  - E. R. Berlekamp, Algebraic Coding Theory, Aegean Park press 1984
  - T. Beth et. al, “Architectures for Exponentiation in  $GF(2^n)$ ,” Crypto 86

# DES in Counter Mode

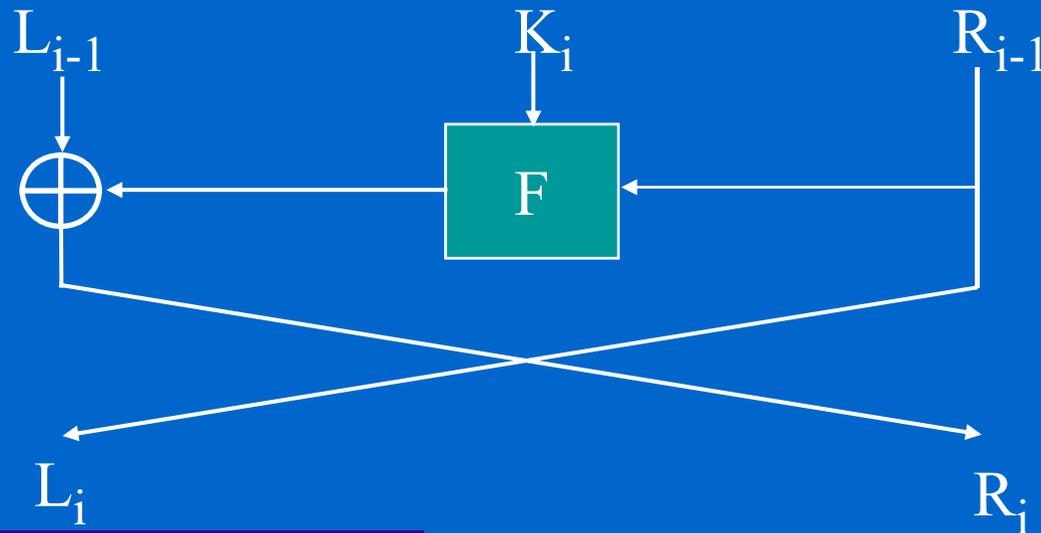


- pseudo one-time pad
- has better security properties than CBC, CFB, OFB encryption modes

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# Feistel Cipher

- Horst Feistel, 1973 IBM LUCIFER
- a common block encryption structure used in many symmetric encryption schemes that maximize the effects of Shannon's "Confusion" and "Diffusion"



# Enigma

- German Enigma cipher machine in World War II. The Enigma had been broken by the Allies in World War II. The capture of the German U-505 submarine in David Kahn's book.
- U-571, 2000 movie; Enigma, 2002 movie
- see John J. G. Savard, A Cryptographic Compendium
  - <http://home.ecn.ab.ca/~jsavard/crypto/entry.htm>
- Codes throughout history
  - <http://codebreaker.dids.com/fhistory.htm>