Chinese Remainder Theorem (CRT)

$$\begin{aligned} n &\equiv r_1 \pmod{m_1} \\ &\equiv r_2 \pmod{m_2} \\ &\quad \bullet\ \bullet\ \bullet \\ &\equiv r_k \pmod{m_k} \end{aligned}$$

$\gcd(m_i, m_j) = 1$

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$\gcd(m_i, m_j) = 1$

- solution:

$m = m_1 m_2 \cdot \cdot \cdot m_k$

$z_i = m / m_i$

$\exists!\ z_i^{-1} \in Z^*_{mi}$ s.t. $z_i \cdot z_i^{-1} \equiv 1 \pmod{m_i}$ (since $\gcd(z_i, m_i) = 1$)

$$n \equiv \sum_{i=1}^{k} z_i \cdot z_i^{-1} \cdot r_i \pmod{m}$$

Chinese Remainder Theorem (CRT)

$$\boxed{\begin{aligned} n &\equiv r_1 \pmod{m_1} \\ &\equiv r_2 \pmod{m_2} \\ &\quad \cdots \\ &\equiv r_k \pmod{m_k} \end{aligned}}$$

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$$z_i = m \,/\, m_i$$

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$$n \equiv \sum_{i=1}^{k} z_i \cdot z_i^{-1} \cdot r_i \pmod{m} \equiv \underbrace{\qquad\qquad\qquad\qquad}_{}$$

k terms

Chinese Remainder Theorem (CRT)

✧ solution:

$$
\begin{aligned}
n &\equiv r_1 \pmod{m_1} \\
&\equiv r_2 \pmod{m_2} \\
&\quad \cdots \\
&\equiv r_k \pmod{m_k}
\end{aligned}
$$

$\gcd(m_i, m_j) = 1$

$$m = m_1\, m_2 \cdot \cdot \cdot\, m_k$$

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$$n \equiv \sum_{i=1}^{k} z_i \cdot z_i^{-1} \cdot r_i \pmod{m} \equiv \underbrace{\blacksquare \qquad\qquad\qquad\qquad}_{\text{k terms}}$$

$$
\begin{aligned}
n &\equiv \mathbf{r_1} \pmod{m_1} \\
&\equiv 0 \pmod{m_2} \\
&\quad \cdots \\
&\equiv 0 \pmod{m_k}
\end{aligned}
$$

Chinese Remainder Theorem (CRT)

$$\begin{aligned} n &\equiv r_1 \pmod{m_1} \\ &\equiv r_2 \pmod{m_2} \\ &\cdots \\ &\equiv r_k \pmod{m_k} \end{aligned}$$

$\gcd(m_i, m_j) = 1$

- solution:

$m = m_1\, m_2 \cdots m_k$

$z_i = m / m_i$

$\exists!\ z_i^{-1} \in Z^*_{mi}$ s.t. $z_i \cdot z_i^{-1} \equiv 1 \pmod{m_i}$ (since $\gcd(z_i, m_i) = 1$)

$$n \equiv \sum_{i=1}^{k} z_i \cdot z_i^{-1} \cdot r_i \pmod{m} \equiv \underbrace{\blacksquare + \blacksquare}_{k\text{ terms}}$$

$$\begin{aligned} n &\equiv 0 \pmod{m_1} \\ &\equiv \mathbf{r_2} \pmod{m_2} \\ &\cdots \\ &\equiv 0 \pmod{m_k} \end{aligned}$$

Chinese Remainder Theorem (CRT)

$$\begin{aligned} n &\equiv r_1 \pmod{m_1} \\ &\equiv r_2 \pmod{m_2} \\ &\;\;\cdots \\ &\equiv r_k \pmod{m_k} \end{aligned}$$

$\gcd(m_i, m_j) = 1$

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$$n \equiv \sum_{i=1}^{k} z_i \cdot z_i^{-1} \cdot r_i \pmod{m} \equiv \overbrace{\blacksquare + \blacksquare + \circ\ \circ\ \circ + \blacksquare}^{k \text{ terms}}$$

$$\begin{aligned} n &\equiv 0 \pmod{m_1} \\ &\equiv 0 \pmod{m_2} \\ &\;\;\cdots \\ &\equiv \mathbf{r_k} \pmod{m_k} \end{aligned}$$

Chinese Remainder Theorem (CRT)

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\boxed{\begin{aligned} n &\equiv r_1 \pmod{m_1} \\ &\equiv r_2 \pmod{m_2} \\ &\cdots \\ &\equiv r_k \pmod{m_k} \end{aligned}}
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- ex: $r_1=1$, $r_2=2$, $r_3=3$

$m_1=3$, $m_2=5$, $m_3=7$ $\qquad m = 3 \cdot 5 \cdot 7$

Chinese Remainder Theorem (CRT)

$$n \equiv r_1 \pmod{m_1}$$
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$$n \equiv \sum_{i=1}^{k} z_i \cdot z_i^{-1} \cdot r_i \pmod{m} \equiv \underbrace{\blacksquare + \blacksquare + \circ\ \circ\ \circ + \blacksquare}_{k \text{ terms}}$$

- ex: $r_1=1$, $r_2=2$, $r_3=3$

$m_1=3$, $m_2=5$, $m_3=7$ $\qquad m = 3 \cdot 5 \cdot 7$

$z_1=35$, $z_2=21$, $z_3=15$

Chinese Remainder Theorem (CRT)

$$n \equiv r_1 \pmod{m_1}$$
$$\equiv r_2 \pmod{m_2}$$
$$\cdots$$
$$\equiv r_k \pmod{m_k}$$

$\gcd(m_i, m_j) = 1$

- solution:

$$m = m_1 m_2 \cdots m_k$$

$$z_i = m / m_i$$

$$\exists!\ z_i^{-1} \in Z^*_{mi} \text{ s.t. } z_i \cdot z_i^{-1} \equiv 1 \pmod{m_i} \text{ (since } \gcd(z_i, m_i) = 1)$$

$$n \equiv \sum_{i=1}^{k} z_i \cdot z_i^{-1} \cdot r_i \pmod{m} \equiv \underbrace{\blacksquare + \blacksquare + \circ\ \circ\ \circ + \blacksquare}_{k \text{ terms}}$$

- ex: $r_1=1,\ r_2=2,\ r_3=3$

$m_1=3,\ m_2=5,\ m_3=7$ $\qquad m = 3 \cdot 5 \cdot 7$

$z_1=35,\ z_2=21,\ z_3=15$

$z_1^{-1}=2,\ z_2^{-1}=1,\ z_3^{-1}=1$ $\qquad 35 \cdot 2 + 3\,(-23) = 1$

Chinese Remainder Theorem (CRT)

$$\boxed{\begin{aligned} n &\equiv r_1 \pmod{m_1} \\ &\equiv r_2 \pmod{m_2} \\ &\quad \cdots \\ &\equiv r_k \pmod{m_k} \end{aligned}}$$

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- ex: $r_1=1,\quad r_2=2,\quad r_3=3$

  $m_1=3, m_2=5, m_3=7 \qquad m = 3 \cdot 5 \cdot 7$

  $z_1=35, z_2=21, z_3=15$

  $z_1^{-1}=2, z_2^{-1}=1, z_3^{-1}=1$

  $n \equiv 35 \cdot 2 \cdot 1 + 21 \cdot 1 \cdot 2 + 15 \cdot 1 \cdot 3 \equiv 157 \equiv 52 \pmod{105}$

CRT, $\gcd(m_1, m_2)=1$

- $n \equiv r_1 \pmod{m_1}$ $\quad\quad$ $\gcd(m_1, m_2) = 1$

  $\equiv r_2 \pmod{m_2}$

CRT, $gcd(m_1, m_2)=1$

- $n \equiv r_1 \pmod{m_1}$
  $\equiv r_2 \pmod{m_2}$

$gcd(m_1, m_2) = 1$

❶這個式子很容易手動算出

- $\exists$ s, t such that $m_1 s + m_2 t = 1$

CRT, gcd(m_1, m_2)=1

- $n \equiv r_1 \pmod{m_1}$
  $\equiv r_2 \pmod{m_2}$

gcd(m_1, m_2) = 1

❶這個式子很容易手動算出

❷中國餘式定理計算時需要的反元素在這個式子裡都有

- $\exists$ s, t such that $m_1 s + m_2 t = 1$

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$\pmod{m_2}$

CRT, $\gcd(m_1, m_2)=1$

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i.e. $m_1 m_1^{-1} + m_2 m_2^{-1} = 1$

$\pmod{m_2}$ $\pmod{m_1}$

CRT, $\gcd(m_1, m_2)=1$

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i.e. $m_1 m_1^{-1} + m_2 m_2^{-1} = 1$

✧ $n \equiv r_1 (m_2 m_2^{-1}) + r_2 (m_1 m_1^{-1}) \pmod{m_1 m_2}$

CRT, $\gcd(m_1, m_2)=1$

- ✧ $n \equiv r_1 \pmod{m_1}$
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CRT, $\gcd(m_1, m_2)=1$

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Verification

CRT, $\gcd(m_1, m_2)=1$

✧ $n \equiv r_1 \pmod{m_1}$
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$\gcd(m_1, m_2) = 1$

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✧ $n \equiv r_1 (m_2 m_2^{-1}) + r_2 (m_1 m_1^{-1}) \pmod{m_1 m_2}$

$n \bmod m_1 =$

Verification

CRT, $\gcd(m_1, m_2)=1$

- $n \equiv r_1 \pmod{m_1}$ $\quad\quad$ $\gcd(m_1, m_2) = 1$

  $\equiv r_2 \pmod{m_2}$

❶這個式子很容易手動算出

❷中國餘式定理計算時需要的反元素在這個式子裡都有

- $\exists\ s, t$ such that $m_1 s + m_2 t = 1$

  i.e. $m_1 m_1^{-1} + m_2 m_2^{-1} = 1$

- $n \equiv r_1 (m_2 m_2^{-1}) + r_2 (m_1 m_1^{-1}) \pmod{m_1 m_2}$

$n \bmod m_1 =$

$n \bmod m_2 =$

Verification

CRT, $\gcd(m_1, m_2)=1$

- $n \equiv r_1 \pmod{m_1}$
  $\equiv r_2 \pmod{m_2}$

$\gcd(m_1, m_2) = 1$

❶這個式子很容易手動算出

❷中國餘式定理計算時需要的反元素在這個式子裡都有

- $\exists\, s, t$ such that $m_1 s + m_2 t = 1$

  i.e. $m_1 m_1^{-1} + m_2 m_2^{-1} = 1$

mod m_1

- $n \equiv r_1 \underbrace{(m_2 m_2^{-1})} + r_2 (m_1 m_1^{-1}) \pmod{m_1 m_2}$

$n \bmod m_1 = r_1$

Verification

CRT, $\gcd(m_1, m_2)=1$

- $n \equiv r_1 \pmod{m_1}$ $\quad\quad \gcd(m_1, m_2) = 1$
  $\equiv r_2 \pmod{m_2}$

❶這個式子很容易手動算出

❷中國餘式定理計算時需要的反元素在這個式子裡都有

- $\exists\ s, t$ such that $m_1 s + m_2 t = 1$

  i.e. $m_1 m_1^{-1} + m_2 m_2^{-1} = 1$

- $n \equiv r_1 \underbrace{(m_2 m_2^{-1})} + r_2 \underbrace{(m_1 m_1^{-1})} \pmod{m_1 m_2}$

$n \bmod m_1 = \quad r_1 \quad + \quad 0$

Verification

CRT, $\gcd(m_1, m_2)=1$

✧ $n \equiv r_1 \pmod{m_1}$
$\quad\equiv r_2 \pmod{m_2}$

$\gcd(m_1, m_2) = 1$

❶這個式子很容易手動算出

❷中國餘式定理計算時需要的反元素在這個式子裡都有

✧ $\exists\ s, t$ such that $m_1 s + m_2 t = 1$

i.e. $m_1 m_1^{-1} + m_2 m_2^{-1} = 1$

mod m_2

✧ $n \equiv r_1 (m_2 m_2^{-1}) + r_2 (m_1 m_1^{-1}) \pmod{m_1 m_2}$

$n \bmod m_1 = \quad r_1 \quad + \quad 0$

$n \bmod m_2 = \quad\quad\quad\quad\ \ r_2$

Verification

CRT, $\gcd(m_1, m_2)=1$

- $n \equiv r_1 \pmod{m_1}$
  $\equiv r_2 \pmod{m_2}$

$\gcd(m_1, m_2) = 1$

❶這個式子很容易手動算出

❷中國餘式定理計算時需要的反元素在這個式子裡都有

- $\exists\ s, t$ such that $m_1 s + m_2 t = 1$

  i.e. $m_1 m_1^{-1} + m_2 m_2^{-1} = 1$

- $n \equiv r_1 \underbrace{(m_2 m_2^{-1})} + r_2 \underbrace{(m_1 m_1^{-1})} \pmod{m_1 m_2}$

$n \bmod m_1 =$	r_1		0	
		$+$		**Verification**
$n \bmod m_2 =$	0		r_2	

Manually Incremental Calculation

$$
\begin{aligned}
n &\equiv \mathbf{1} \pmod{3} \\
  &\equiv \mathbf{2} \pmod{5} \\
  &\equiv \mathbf{3} \pmod{7}
\end{aligned}
$$

Manually Incremental Calculation

$$\begin{aligned} n &\equiv \mathbf{1} \pmod 3 \\ &\equiv \mathbf{2} \pmod 5 \\ &\equiv \mathbf{3} \pmod 7 \end{aligned}$$

① $\hat{n}_1 \equiv \underset{r_1}{\mathbf{1}} \pmod 3$ … satisfying the 1st eq.

Manually Incremental Calculation

$$\begin{aligned} n &\equiv 1 \pmod 3 \\ &\equiv 2 \pmod 5 \\ &\equiv 3 \pmod 7 \end{aligned} \qquad\qquad \begin{aligned} n &\equiv 1 \pmod 3 \\ &\equiv 2 \pmod 5 \end{aligned}$$

① $\hat{n}_1 \equiv 1 \pmod 3$ … satisfying the 1st eq.

Manually Incremental Calculation

$$n \equiv 1 \pmod 3 \qquad\qquad n \equiv 1 \pmod 3$$
$$\equiv 2 \pmod 5 \qquad\qquad \equiv 2 \pmod 5$$
$$\equiv 3 \pmod 7$$

① $\hat{n}_1 \equiv 1 \pmod 3$ … satisfying the 1st eq.

② $3 \cdot (-3) + 5 \cdot 2 = 1$

Manually Incremental Calculation

$$n \equiv \mathbf{1} \pmod{3}$$
$$\equiv \mathbf{2} \pmod{5}$$
$$\equiv \mathbf{3} \pmod{7}$$

$$n \equiv \mathbf{1} \pmod{3}$$
$$\equiv \mathbf{2} \pmod{5}$$

① $\hat{n}_1 \equiv \mathbf{1} \pmod{3}$ … satisfying the 1^{st} eq.

inverse of 3 (mod 5)

② $3 \cdot (-3) + 5 \cdot 2 = 1$

Manually Incremental Calculation

$n \equiv 1 \pmod 3$
$\equiv 2 \pmod 5$
$\equiv 3 \pmod 7$

$n \equiv 1 \pmod 3$
$\equiv 2 \pmod 5$

① $\hat{n}_1 \equiv 1 \pmod 3$ … satisfying the 1st eq.

inverse of 3 (mod 5)

② $3 \cdot (-3) + 5 \cdot 2 = 1$

inverse of 5 (mod 3)

Manually Incremental Calculation

$$n \equiv 1 \pmod 3 \qquad\qquad n \equiv 1 \pmod 3$$
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$$\equiv 3 \pmod 7$$

① $\hat{n}_1 \equiv 1 \pmod 3$ … satisfying the 1st eq.

inverse of 3 (mod 5)

② $3 \cdot (-3) + 5 \cdot 2 = 1$

inverse of 5 (mod 3)

③ $\hat{n}_2 \equiv 2 \cdot 3 \cdot (-3) + \underbrace{1}_{\hat{n}_1} \cdot 5 \cdot 2$

Manually Incremental Calculation

$$n \equiv 1 \pmod 3$$
$$\equiv 2 \pmod 5$$
$$\equiv 3 \pmod 7$$

$$n \equiv 1 \pmod 3$$
$$\equiv 2 \pmod 5$$

① $\hat{n}_1 \equiv 1 \pmod 3$ … satisfying the 1st eq.

inverse of 3 (mod 5)

② $3 \cdot (-3) + 5 \cdot 2 = 1$

inverse of 5 (mod 3)

③ $\hat{n}_2 \equiv \underbrace{2}_{r_2} \cdot 3 \cdot (-3) + \underbrace{1}_{\hat{n}_1} \cdot 5 \cdot 2$

Manually Incremental Calculation

$$n \equiv 1 \pmod{3}$$
$$\equiv 2 \pmod{5}$$
$$\equiv 3 \pmod{7}$$

$$n \equiv 1 \pmod{3}$$
$$\equiv 2 \pmod{5}$$

① $\hat{n}_1 \equiv 1 \pmod{3}$ … satisfying the 1st eq.

② $3 \cdot (-3) + 5 \cdot 2 = 1$

③ $\hat{n}_2 \equiv 2 \cdot 3 \cdot (-3) + 1 \cdot 5 \cdot 2 \equiv -8 \equiv 7 \pmod{15}$ …. satisfying first 2 eqs.

Manually Incremental Calculation

$$\begin{aligned} n &\equiv 1 \pmod 3) \\ &\equiv 2 \pmod 5 \\ &\equiv 3 \pmod 7 \end{aligned}$$

$$\begin{aligned} n &\equiv 7 \pmod{15} \\ &\equiv 3 \pmod 7 \end{aligned}$$

① $\hat{n}_1 \equiv 1 \pmod 3$ … satisfying the 1st eq.

② $3 \cdot (-3) + 5 \cdot 2 = 1$

③ $\hat{n}_2 \equiv 2 \cdot 3 \cdot (-3) + 1 \cdot 5 \cdot 2 \equiv -8 \equiv 7 \pmod{15}$ …. satisfying first 2 eqs.

Manually Incremental Calculation

$$n \equiv 1 \pmod{3}$$
$$\equiv 2 \pmod{5}$$
$$\equiv 3 \pmod{7}$$

$$n \equiv 7 \pmod{15}$$
$$\equiv 3 \pmod{7}$$

① $\hat{n}_1 \equiv 1 \pmod{3}$ … satisfying the 1st eq.

② $3 \cdot (-3) + 5 \cdot 2 = 1$

③ $\hat{n}_2 \equiv 2 \cdot 3 \cdot (-3) + 1 \cdot 5 \cdot 2 \equiv -8 \equiv 7 \pmod{15}$ …. satisfying first 2 eqs.

④ $15 \cdot 1 + 7 \cdot (-2) = 1$

Manually Incremental Calculation

$n \equiv \mathbf{1} \pmod 3$
$\equiv \mathbf{2} \pmod 5$
$\equiv \mathbf{3} \pmod 7$

$n \equiv \mathbf{7} \pmod{15}$
$\equiv \mathbf{3} \pmod 7$

① $\hat{n}_1 \equiv \mathbf{1} \pmod 3$ … satisfying the 1st eq.

② $3 \cdot (-3) + 5 \cdot 2 = 1$

③ $\hat{n}_2 \equiv \mathbf{2} \cdot 3 \cdot (-3) + \mathbf{1} \cdot 5 \cdot 2 \equiv -8 \equiv \mathbf{7} \pmod{15}$ …. satisfying first 2 eqs.

inverse of 15 (mod 7)

④ $15 \cdot 1 + 7 \cdot (-2) = 1$

inverse of 7 (mod 15)

Manually Incremental Calculation

$n \equiv 1 \pmod 3$
$\equiv 2 \pmod 5$
$\equiv 3 \pmod 7$

$n \equiv 7 \pmod{15}$
$\equiv 3 \pmod 7$

① $\hat{n}_1 \equiv 1 \pmod 3$ … satisfying the 1^{st} eq.

② $3 \cdot (-3) + 5 \cdot 2 = 1$

③ $\hat{n}_2 \equiv 2 \cdot 3 \cdot (-3) + 1 \cdot 5 \cdot 2 \equiv -8 \equiv 7 \pmod{15}$ …. satisfying first 2 eqs.

inverse of 15 (mod 7)

④ $15 \cdot 1 + 7 \cdot (-2) = 1$

inverse of 7 (mod 15)

⑤ $\hat{n}_3 \equiv 3 \cdot 15 \cdot 1 + \underbrace{7}_{\hat{n}_2} \cdot 7 \cdot (-2)$

Manually Incremental Calculation

$n \equiv 1 \pmod 3$
$\equiv 2 \pmod 5$
$\equiv 3 \pmod 7$

$n \equiv 7 \pmod{15}$
$\equiv 3 \pmod 7$

① $\hat{n}_1 \equiv 1 \pmod 3$ … satisfying the 1st eq.

② $3 \cdot (-3) + 5 \cdot 2 = 1$

③ $\hat{n}_2 \equiv 2 \cdot 3 \cdot (-3) + 1 \cdot 5 \cdot 2 \equiv -8 \equiv 7 \pmod{15}$ …. satisfying first 2 eqs.

inverse of 15 (mod 7)

④ $15 \cdot 1 + 7 \cdot (-2) = 1$

inverse of 7 (mod 15)

⑤ $\hat{n}_3 \equiv \underset{r_3}{3} \cdot 15 \cdot 1 + \underset{\hat{n}_2}{7} \cdot 7 \cdot (-2)$

Manually Incremental Calculation

$$n \equiv 1 \pmod 3$$
$$\equiv 2 \pmod 5$$
$$\equiv 3 \pmod 7$$

$$n \equiv 7 \pmod{15}$$
$$\equiv 3 \pmod 7$$

① $\hat{n}_1 \equiv 1 \pmod 3$ … satisfying the 1st eq.

② $3 \cdot (-3) + 5 \cdot 2 = 1$

③ $\hat{n}_2 \equiv 2 \cdot 3 \cdot (-3) + 1 \cdot 5 \cdot 2 \equiv -8 \equiv 7 \pmod{15}$ …. satisfying first 2 eqs.

④ $15 \cdot 1 + 7 \cdot (-2) = 1$

⑤ $\hat{n}_3 \equiv 3 \cdot 15 \cdot 1 + 7 \cdot 7 \cdot (-2) \equiv -53 \equiv \mathbf{52} \pmod{105}$ … satisfying all 3 eqs.

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CRT, gcd(m_1, m_2)=d

- $n \equiv r_1 \pmod{m_1}$ **moduli are not relative prime**
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Verification: 26 mod 6 = **2**, 26 mod 10 = **6**

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  $\equiv r_2 \pmod{m_2}$ gcd(m_1, m_2) = **d** > 1

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CRT

$n \equiv 1 \pmod 6 \;\Leftrightarrow\; n \equiv 1 \pmod 2 \equiv 1 \pmod 3$ $\quad \gcd(2,3)=1$

$n \equiv 3 \pmod{10} \;\Leftrightarrow\; n \equiv 1 \pmod 2 \equiv 3 \pmod 5$ $\quad \gcd(2,5)=1$

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$$n \equiv 1 \pmod 6 \overset{\text{CRT}}{\Leftrightarrow} n \equiv 1 \pmod 2 \equiv 1 \pmod 3$$

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consistent

$\gcd(2,5)=1$

CRT, $\gcd(m_1, m_2)=d$

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Verification: $26 \bmod 6 = \cancel{2}$, $26 \bmod 10 = \cancel{6}$ Incorrect!!!

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$n \equiv 1 \pmod 2$

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Verification: 26 mod 6 = $\cancel{2}$, 26 mod 10 = $\cancel{6}$ Incorrect!!!

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$$n \equiv 3 \pmod{10} \Leftrightarrow n \equiv 1 \pmod{2} \equiv 3 \pmod{5}$$

$$\left.\begin{aligned} n &\equiv 1 \pmod{2} \\ &\equiv 1 \pmod{3} \\ &\equiv 3 \pmod{5} \end{aligned}\right\} \Rightarrow \begin{aligned} n &\equiv 1 \pmod{6} \\ &\equiv 3 \pmod{5} \end{aligned}$$

i.e. $\boxed{\begin{aligned} n &\equiv r_1 \pmod{m_1} \\ &\equiv r_2 \pmod{m_2/d} \end{aligned}}$

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Verification: $26 \bmod 6 = \cancel{2}$, $26 \bmod 10 = \cancel{6}$ Incorrect!!!

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$$\left.\begin{array}{lcl} n \equiv 1 \pmod 6 & \overset{\text{CRT}}{\Leftrightarrow} & n \equiv \mathbf{1} \pmod{\mathbf{2}} \equiv 1 \pmod 3 \\ n \equiv 3 \pmod{10} & \Leftrightarrow & n \equiv \mathbf{1} \pmod{\mathbf{2}} \equiv 3 \pmod 5 \end{array}\right\}$$

note: CRT works only when $\gcd(d,m_2/d)=1$

$$\left.\begin{array}{l} n \equiv 1 \pmod 2 \\ \quad \equiv 1 \pmod 3 \end{array}\right\} \Rightarrow \begin{array}{l} n \equiv 1 \pmod 6 \\ \quad \equiv 3 \pmod 5 \end{array}$$
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i.e. $\boxed{\begin{array}{l} n \equiv r_1 \pmod{m_1} \\ \quad \equiv r_2 \pmod{m_2/\mathbf{d}} \end{array}}$

CAVEAT

- $$\begin{aligned} n &\equiv 3 \pmod{10} \\ &\equiv 11 \pmod{12} \end{aligned}$$

CAVEAT

$10=2\cdot 5,\ 12=2^2\cdot 3$

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$10=2\cdot 5,\ 12=2^2\cdot 3 \qquad \gcd(10,12)=2$

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$53 \pmod{60}$

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$\cancel{53 \pmod{60}}$

CAVEAT

$10=2\cdot 5,\ 12=2^2\cdot 3$ $\quad$ $\gcd(10,12)=2$ $\quad$ $\gcd(10,6)=2$

- $n \equiv 3 \pmod{10}$
  $\equiv 11 \pmod{12}$

$\Rightarrow$ $n \equiv 3 \pmod{10}$, $\cancel{\equiv 5 \pmod{6}}$ $\Rightarrow$ $n \equiv 3 \pmod{10}$, $\equiv 2 \pmod{3}$

$\cancel{53 \pmod{60}}$ $\Rightarrow$ $n \equiv 23 \pmod{30}$

CAVEAT

$10=2\cdot 5,\ 12=2^2\cdot 3 \qquad \gcd(10,12)=2 \qquad \gcd(10,6)=2$

- $n \equiv 3 \pmod{10}$
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$12=2^2\cdot3$

gcd(4,3)=1

$$n \equiv 11 \pmod{12} \quad \overset{\text{CRT}}{\Longleftrightarrow} \quad n \equiv 3 \pmod 4 \equiv 2 \pmod 3$$

CAVEAT

$10=2\cdot 5,\ 12=2^2\cdot 3$ $\quad \gcd(10,12)=2$ $\quad \gcd(10,6)=2$

✧ $n \equiv 3 \pmod{10}$ $\equiv 11 \pmod{12}$ $\Rightarrow$ $n \equiv 3 \pmod{10}$ $\cancel{\equiv 5 \pmod 6}$ $\Rightarrow$ $\cancel{n \equiv 3 \pmod{10} \equiv 2 \pmod 3}$ $\Rightarrow$ $\cancel{n \equiv 23 \pmod{30}}$

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$12=2^2\cdot 3$

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$n \equiv 11 \pmod{12} \quad \cancel{\Leftrightarrow} \quad n \equiv 1 \pmod 2 \equiv 5 \pmod 6$

gcd(2,6)≠1

CAVEAT

$10 = 2\cdot5,\ 12 = 2^2\cdot3 \quad \gcd(10,12)=2 \quad \gcd(10,6)=2$

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$n \equiv 1 \pmod 2 \equiv 5 \pmod 6$

CAVEAT

$10=2\cdot 5,\ 12=2^2\cdot 3 \qquad \gcd(10,12)=2 \qquad \gcd(10,6)=2$

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$12=2^2\cdot 3$

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$$n \equiv 11 \pmod{12} \quad \cancel{\Longleftrightarrow} \quad n \equiv 1 \pmod{2} \equiv 5 \pmod{6}$$

$$n \equiv 1 \pmod{2} \equiv 5 \pmod{6} \ \Longleftrightarrow\ n \equiv 1 \pmod{2} \equiv 1 \pmod{2} \equiv 2 \pmod{3}$$

CAVEAT

$10=2\cdot 5,\ 12=2^2\cdot 3 \qquad \gcd(10,12)=2 \qquad \gcd(10,6)=2$

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$12=2^2\cdot 3$

$$n \equiv 11 \pmod{12} \overset{\text{CRT}}{\Leftrightarrow} n \equiv 3 \pmod{4} \equiv 2 \pmod{3}$$

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CAVEAT

$10=2\cdot 5,\ 12=2^2\cdot 3$ $\quad \gcd(10,12)=2$ $\quad \gcd(10,6)=2$

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  $\equiv 11 \pmod{12}$ $\Rightarrow$ $n \equiv 3 \pmod{10}$, $\cancel{\equiv 5 \pmod 6}$ $\Rightarrow$ $\cancel{n \equiv 3 \pmod{10},\ \equiv 2 \pmod 3}$

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$$n \equiv 1 \pmod 2 \equiv 5 \pmod 6 \Leftrightarrow n \equiv 1 \pmod 2 \equiv 1 \pmod 2 \equiv 2 \pmod 3$$
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$\cancel{53 \pmod{60}}$ $\Rightarrow$ $\cancel{n \equiv 23 \pmod{30}}$

$12=2^2\cdot 3$

$$n \equiv 11 \pmod{12} \quad \overset{\text{CRT}}{\Leftrightarrow} \quad n \equiv 3 \pmod 4 \equiv 2 \pmod 3$$

$$n \equiv 11 \pmod{12} \quad \cancel{\Leftrightarrow} \quad n \equiv 1 \pmod 2 \equiv 5 \pmod 6$$

$$n \equiv 1 \pmod 2 \equiv 5 \pmod 6 \Leftrightarrow n \equiv 1 \pmod 2 \equiv 1 \pmod 2 \equiv 2 \pmod 3$$
$$\Leftrightarrow n \equiv 1 \pmod 2 \equiv 2 \pmod 3$$
$$\Leftrightarrow n \equiv 5 \pmod 6$$

✧ $n \equiv 3 \pmod{10}$
$\quad \equiv 11 \pmod{12}$

CAVEAT

$10=2\cdot 5,\ 12=2^2\cdot 3 \qquad \gcd(10,12)=2 \qquad \gcd(10,6)=2$

✧ $n \equiv 3 \pmod{10} \equiv 11 \pmod{12} \Rightarrow n \equiv 3 \pmod{10}$, $\cancel{\equiv 5 \pmod{6}} \Rightarrow \cancel{n \equiv 3 \pmod{10} \equiv 2 \pmod{3}}$; $\cancel{53 \pmod{60}} \Rightarrow \cancel{n \equiv 23 \pmod{30}}$

$12=2^2\cdot 3$

$$n \equiv 11 \pmod{12} \overset{\text{CRT}}{\Leftrightarrow} n \equiv 3 \pmod{4} \equiv 2 \pmod{3}$$

$$n \equiv 11 \pmod{12} \ \cancel{\Leftrightarrow}\ n \equiv 1 \pmod{2} \equiv 5 \pmod{6}$$

$$\begin{aligned} n \equiv 1 \pmod{2} \equiv 5 \pmod{6} &\Leftrightarrow n \equiv 1 \pmod{2} \equiv 1 \pmod{2} \equiv 2 \pmod{3} \\ &\Leftrightarrow n \equiv 1 \pmod{2} \equiv 2 \pmod{3} \\ &\Leftrightarrow n \equiv 5 \pmod{6} \end{aligned}$$

✧ $n \equiv 3 \pmod{10} \equiv 11 \pmod{12} \Rightarrow n \equiv 1 \pmod{2} \equiv 3 \pmod{5} \equiv 3 \pmod{4} \equiv 2 \pmod{3}$

CAVEAT

$10=2\cdot5,\ 12=2^2\cdot3 \quad \gcd(10,12)=2 \quad \gcd(10,6)=2$

✧ $n \equiv 3 \pmod{10}$
$\equiv 11 \pmod{12}$

$\Rightarrow$ $n \equiv 3 \pmod{10}$
$\cancel{\equiv 5 \pmod 6}$

$\Rightarrow$ $\cancel{n \equiv 3 \pmod{10}}$
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✧ $n \equiv 3 \pmod{10}$
$\equiv 11 \pmod{12}$

$\Rightarrow$ $n \equiv 1 \pmod 2$
$\equiv 3 \pmod 5$
$\equiv 3 \pmod 4$
$\equiv 2 \pmod 3$

$\Rightarrow$ $n \equiv 3 \pmod 5$
$\equiv 3 \pmod 4$
$\equiv 2 \pmod 3$

CAVEAT

$10=2\cdot 5,\ 12=2^2\cdot 3 \qquad \gcd(10,12)=2 \qquad \gcd(10,6)=2$

✧ $n \equiv 3 \pmod{10}$
$\equiv 11 \pmod{12}$ $\Rightarrow$ $n \equiv 3 \pmod{10}$
$\cancel{\equiv 5 \pmod{6}}$ $\Rightarrow$ $\cancel{n \equiv 3 \pmod{10}}$
$\cancel{\equiv 2 \pmod{3}}$ $\Rightarrow$ $\cancel{n \equiv 23 \pmod{30}}$

$\cancel{53 \pmod{60}}$

$12=2^2\cdot 3$

$n \equiv 11 \pmod{12} \overset{\text{CRT}}{\Leftrightarrow} n \equiv 3 \pmod{4} \equiv 2 \pmod{3}$

$n \equiv 11 \pmod{12}\ \cancel{\Leftrightarrow}\ n \equiv 1 \pmod{2} \equiv 5 \pmod{6}$

$n \equiv 1 \pmod{2} \equiv 5 \pmod{6} \Leftrightarrow n \equiv 1 \pmod{2} \equiv 1 \pmod{2} \equiv 2 \pmod{3}$
$\Leftrightarrow n \equiv 1 \pmod{2} \equiv 2 \pmod{3}$
$\Leftrightarrow n \equiv 5 \pmod{6}$

✧ $n \equiv 3 \pmod{10}$
$\equiv 11 \pmod{12}$ $\Rightarrow$ $n \equiv 1 \pmod{2}$
$\equiv 3 \pmod{5}$
$\equiv 3 \pmod{4}$
$\equiv 2 \pmod{3}$
$\Rightarrow$ $n \equiv 3 \pmod{5}$
$\equiv 3 \pmod{4}$
$\equiv 2 \pmod{3}$
$\Rightarrow$ $n \equiv 3 \pmod{20}$
$\equiv 2 \pmod{3}$

CAVEAT

$10=2\cdot 5,\ 12=2^2\cdot 3 \qquad \gcd(10,12)=2 \qquad \gcd(10,6)=2$

✧ $$\begin{aligned} n &\equiv 3 \pmod{10} \\ &\equiv 11 \pmod{12} \end{aligned} \;\Rightarrow\; \begin{aligned} n &\equiv 3 \pmod{10} \\ &\cancel{\equiv 5 \pmod{6}} \end{aligned} \;\Rightarrow\; \cancel{\begin{aligned} n &\equiv 3 \pmod{10} \\ &\equiv 2 \pmod{3} \end{aligned}} \;\Rightarrow\; \cancel{n \equiv 23 \pmod{30}}$$

$$\cancel{53 \pmod{60}}$$

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$$\begin{aligned} n \equiv 1 \pmod{2} \equiv 5 \pmod{6} &\Leftrightarrow n \equiv 1 \pmod{2} \equiv 1 \pmod{2} \equiv 2 \pmod{3} \\ &\Leftrightarrow n \equiv 1 \pmod{2} \equiv 2 \pmod{3} \\ &\Leftrightarrow n \equiv 5 \pmod{6} \end{aligned}$$

✧ $$\begin{aligned} n &\equiv 3 \pmod{10} \\ &\equiv 11 \pmod{12} \end{aligned} \;\Rightarrow\; \begin{aligned} n &\equiv 1 \pmod{2} \\ &\equiv 3 \pmod{5} \\ &\equiv 3 \pmod{4} \\ &\equiv 2 \pmod{3} \end{aligned} \;\Rightarrow\; \begin{aligned} n &\equiv 3 \pmod{5} \\ &\equiv 3 \pmod{4} \\ &\equiv 2 \pmod{3} \end{aligned} \;\Rightarrow\; \begin{aligned} n &\equiv 3 \pmod{20} \\ &\equiv 2 \pmod{3} \end{aligned} \;\Rightarrow\; \boxed{n \equiv 23 \pmod{60}}$$

CRT w/ Moluli not Relative Prime

- **Chinese Remainder** Theorem:

CRT w/ Moluli not Relative Prime

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$$
\begin{aligned}
\mathbf{n} &\equiv r_1 \pmod{m_1} \\
&\equiv r_2 \pmod{m_2} \\
&\quad \cdots \\
&\equiv r_k \pmod{m_k}
\end{aligned}
$$

CRT w/ Moluli not Relative Prime

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$\gcd(m_i, m_j) = 1$

CRT w/ Moluli not Relative Prime

- **Chinese Remainder** Theorem:

  there exists a unique integer
  n $\in Z_{m_1 \cdots m_k}$ satisfying the
  set of k congruence equations

$$\boxed{\begin{aligned} \mathbf{n} &\equiv r_1 \pmod{m_1} \\ &\equiv r_2 \pmod{m_2} \\ &\quad \bullet\bullet\bullet \\ &\equiv r_k \pmod{m_k} \end{aligned}}$$

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note: each tuple $(r_1, r_2 \cdots, r_k)$ maps to one distinct integer in $[0, m_1 m_2 \cdots m_k - 1]$, which are members of the field $Z_{m_1 \cdots m_k}$

CRT w/ Moluli not Relative Prime

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- **Prime power moduli**: $n \equiv r \pmod{p^c}$

CRT w/ Moluli not Relative Prime

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CRT w/ Moluli not Relative Prime

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- **CRT** with **prime modulus**: $n \equiv r \pmod{m}$

CRT w/ Moluli not Relative Prime

- **Chinese Remainder** Theorem:

  there exists a unique integer

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$$\begin{aligned} \mathbf{n} &\equiv r_1 \pmod{m_1} \\ &\equiv r_2 \pmod{m_2} \\ &\cdots \\ &\equiv r_k \pmod{m_k} \end{aligned}$$

$\gcd(m_i, m_j) = 1$

note: each tuple $(r_1, r_2 \cdots, r_k)$ maps to one distinct integer in $[0, m_1 m_2 \cdots m_k - 1]$, which are members of the field $Z_{m_1 \cdots m_k}$

- **Prime power moduli**: $n \equiv r \pmod{p^c}$

$$\Rightarrow n \equiv r' \pmod{p^{c'}},\ \forall c' < c,\ r' \equiv r \pmod{p^{c'}}$$

- **CRT** with **prime modulus**: $n \equiv r \pmod{m}$

$$m = p_1^{c1} p_2^{c2} \cdots p_k^{ck}$$

Unique Prime Factorization Theorem

CRT w/ Moluli not Relative Prime

- **Chinese Remainder** Theorem:

  there exists a unique integer $\mathbf{n} \in Z_{m_1 \cdots m_k}$ satisfying the set of k congruence equations

$$\boxed{\begin{aligned} \mathbf{n} &\equiv r_1 \pmod{m_1} \\ &\equiv r_2 \pmod{m_2} \\ &\bullet\bullet\bullet \\ &\equiv r_k \pmod{m_k} \end{aligned}}$$

$\gcd(m_i, m_j) = 1$

note: each tuple $(r_1, r_2 \cdots, r_k)$ maps to one distinct integer in $[0, m_1 m_2 \cdots m_k - 1]$, which are members of the field $Z_{m_1 \cdots m_k}$

- **Prime power moduli**: $n \equiv r \pmod{p^c}$

$$\Rightarrow n \equiv r' \pmod{p^{c'}},\ \forall c' < c,\ r' \equiv r \pmod{p^{c'}}$$

- **CRT** with **prime modulus**: $n \equiv r \pmod{m}$ $\Leftrightarrow$ $\boxed{\begin{aligned} n &\equiv r_1 \pmod{p_1^{c_1}} \\ &\equiv r_2 \pmod{p_2^{c_2}} \\ &\bullet\bullet\bullet \\ &\equiv r_k \pmod{p_k^{c_k}} \end{aligned}}$

$$m = p_1^{c1} p_2^{c2} \cdots p_k^{ck}$$

Unique Prime Factorization Theorem

CRT w/ Moluli not Relative Prime

- **CRT** with **moduli not relative prime**:

CRT w/ Moluli not Relative Prime

✧ **CRT** with **moduli not relative prime**:

$n \equiv r_1 \pmod{m_1}$

CRT w/ Moluli not Relative Prime

- **CRT** with **moduli not relative prime**:

$$n \equiv r_1 \pmod{m_1} \quad m_1 = p_1^{c_1} p_2^{c_2} \cdots p_s^{c_s}$$

CRT w/ Moluli not Relative Prime

- **CRT** with **moduli not relative prime**:

$$\begin{cases} n \equiv r_1 \pmod{m_1} & m_1 = p_1^{c_1} p_2^{c_2} \cdots p_s^{c_s} \\ \\ n \equiv r_2 \pmod{m_2} \end{cases}$$

CRT w/ Moluli not Relative Prime

- **CRT** with **moduli not relative prime**:

$$\begin{cases} n \equiv r_1 \pmod{m_1} & m_1 = p_1^{c_1} p_2^{c_2} \cdots p_s^{c_s} \\ n \equiv r_2 \pmod{m_2} & m_2 = q_1^{d_1} q_2^{d_2} \cdots\cdots q_t^{d_t} \end{cases}$$

CRT w/ Moluli not Relative Prime

- **CRT** with **moduli not relative prime**:

$$\begin{cases} n \equiv r_1 \pmod{m_1} & m_1 = p_1^{c_1} p_2^{c_2} \cdots p_s^{c_s} \\ n \equiv r_2 \pmod{m_2} & m_2 = q_1^{d_1} q_2^{d_2} \cdots\cdots q_t^{d_t} \end{cases}$$

$\exists$ i, j, such that $p_i = q_j$

i.e. mululi share common factors

CRT w/ Moluli not Relative Prime

- **CRT** with **moduli not relative prime**:

$$\begin{cases} n \equiv r_1 \pmod{m_1} & m_1 = p_1^{c_1} p_2^{c_2} \cdots p_s^{c_s} \\ n \equiv r_2 \pmod{m_2} & m_2 = q_1^{d_1} q_2^{d_2} \cdots\cdots q_t^{d_t} \end{cases} \Leftrightarrow \left\{ \boxed{\begin{aligned} n &\equiv r_{11} \pmod{p_1^{c_1}} \\ &\equiv r_{12} \pmod{p_2^{c_2}} \\ &\quad \cdots \\ &\equiv r_{1s} \pmod{p_s^{c_s}} \end{aligned}} \right.$$

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CRT w/ Moluli not Relative Prime

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$\exists$ i, j, such that $p_i = q_j$

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CRT w/ Moluli not Relative Prime

- **CRT** with **moduli not relative prime**:

$$\begin{cases} n \equiv r_1 \pmod{m_1} \quad m_1 = p_1^{c_1} p_2^{c_2} \cdots p_s^{c_s} \\ n \equiv r_2 \pmod{m_2} \quad m_2 = q_1^{d_1} q_2^{d_2} \cdots\cdots q_t^{d_t} \end{cases} \Longleftrightarrow \begin{cases} \boxed{\begin{aligned} n &\equiv r_{11} \pmod{p_1^{c_1}} \\ &\equiv r_{12} \pmod{p_2^{c_2}} \\ &\cdots \\ &\equiv r_{1s} \pmod{p_s^{c_s}} \end{aligned}} \\ \boxed{\begin{aligned} n &\equiv r_{21} \pmod{q_1^{d_1}} \\ &\equiv r_{22} \pmod{q_2^{d_2}} \\ &\cdots \\ &\equiv r_{2t} \pmod{q_t^{d_t}} \end{aligned}} \end{cases}$$

$\exists$ i, j, such that $p_i = q_j$
i.e. mululi share common factors

solution exists if $\mathbf{r_{1i} \equiv r_{2j} \pmod{p_i^k}}$, for $p_i = q_j$, $\mathbf{k} = \min(c_i,d_j)$