

# Classical Cryptography

- Monoalphabetic ciphers: letters of the plaintext alphabet are mapped into unique ciphertext letters
- Polyalphabetic ciphers: letters of the plaintext alphabet are mapped into ciphertext letters depending on the context of the plaintext
- Stream ciphers: a key stream is generated and used to encrypt the plaintext

# Shift Cipher

- *Caesar Cipher* : shift cipher with k = 3
- **Example:** Let the key k = 17
  - Plaintext: X = a t t a c k = (0, 19, 19, 0, 2, 10)
  - Ciphertext : Y = (0+17 mod 26, 19+17 mod 26, ...) = (17, 10, 10, 17, 19, 1) = R K K R T B
- Attacks
  - Ciphertext only:
    - Exhaustive Search: Try all possible keys. /K/=26. Nowadays, for moderate security  $/K/ \ge 2^{80}$ , for recommended security  $/K/ \ge 2^{100}$ .
    - Letter frequency analysis (Same plaintext maps to same ciphertext

# Frequency Analysis

• In most languages, letters occur in texts with different frequencies

•	single, double, triple letter frequencies								
	Single	Frequency	Double	Triple					
	Е	.127	TH	THE					
	Т	.091	HE	ING					
	А	.082	IN	AND					
	0	.075	ER	HER					
	Ι	.070	AN	ERE					
	Ν	.067	RE	ENT					
	S	.063	ED	THA					
	Н	.061	ON	NTH					

# Shift Cipher

- Known plaintext: You can deduce the key if you know one letter of the plaintext along with its corresponding ciphertext. Ex. t(=19) encrypts to D(=3), then the key is k = 3 19 = -16 = 10 (mod 26)
- Chosen plaintext: choose the letter 'a' as the plaintext, the ciphertext is the key
- Chosen ciphertext: choose the letter 'A' as ciphertext, the plaintext is the negative of the key

# Letter Frequency Analysis

- Method 1: Find the most frequent cipher character, make a guess as  $E_k$ ('e'), solves k. Use this k to decrypt ciphertext and see if it is a reasonable guess. Otherwise, find the second frequent cipher character, make a guess as  $E_k$ ('e').
- Method 2: correlation
   A<sub>0</sub>=[.082 .015 .028 .043 .127 .022 .020 .061 .070 .002 .008 .040 .024 .067 .075 .019 .001 .060 .063 .091 .028 .010 .023 .001 .020 .001]
   A is obtained by circularly shift right A is elements
  - $A_i$  is obtained by circularly shift right  $A_0$  i elements e.g.  $A_2$ =[.020 .001 .082 .015 .028 .043 ...
- correlation =  $A_i \cdot A_i$  is the usual dot product between  $A_i$  and  $A_i$
- let A be the frequency of the ciphertext paragraph
- calculate correlation between A and A<sub>i</sub>, choose the maximum

# Shift Cipher

- One time pad can be considered as a shift cipher with modulus 2 and a changing key sequence, in which each key is used only for one plaintext character and never repeated.
- A shift cipher as defined is therefore perfectly secure if the key keeps changing and is used for one character only.

# Matlab Example

- dir, cd, help
- path(path, 'c:\lcwMatlabCode')
- k = 20

plain = 'hellothisisashiftcipherexample'
plain\_i = text2int(plain)
cipher\_i = mod(plain\_i + k, 26)
cipher = int2text(cipher\_i)
recovered\_i = mod(cipher\_i - k, 26)
recovered = int2text(recovered\_i)

 cipher = shift(plain, k) recovered = shift(cipher, -k)

## Affine Cipher

- Algorithm: Let  $P = C = Z_{26}$  and  $x \in P$ ,  $Y \in C$ 
  - *Encryption*:  $E_k(x) = Y = \alpha \cdot x + \beta \mod 26$
  - The key  $k = (\alpha, \beta)$  and  $\alpha, \beta \in \mathbb{Z}_{26}$
  - $ex. \alpha=13, \beta=4$

input = (8, 13, 15, 20, 19)	$\Rightarrow (4, 17, 17, 4, 17) = \text{ERRER}$
alter = (0, 11, 19, 4, 17)	$\Rightarrow (4, 17, 17, 4, 17) = \text{ERRER}$

 There is no one-to-one mapping between plaintext and ciphertext. What's wrong?

- Decryption: 
$$D_k(Y) = x = \alpha^{-1} \cdot (Y - \beta)$$
  
=  $\alpha' \cdot Y + \beta' \mod 26$ 

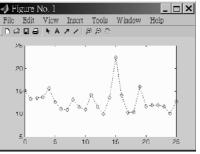
# Matlab letter frequency analysis

#### • sci=

['themethodusedforthepreparationandreadingofcodemessagesissimplei', ... 'ntheextremeandatthesametimeimpossibleoftranslationunlessthekeyi', ... 'sknowntheeasewithwhichthekeymaybechangedisanotherpointinfavorof', ... 'theadoptionofthiscodebythosedesiringtotransmitimportantmessages', ... 'withouttheslightestdangeroftheirmessagesbeingreadbypoliticalorb', ...

'usinessrivalsetc'];

- cipher=shift(sci, 15);
- freq=frequency(cipher);
- correlation=corr(freq);
- plot(0:25,correlation,'bd:')



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# Affine Cipher

#### • Key Space:

- $-\beta$  can be any number in  $Z_{26}$ . 26 possibilities
- Since α<sup>-1</sup> is required to exist, we can only select integers in Z<sub>26</sub> s.t. gcd(α, 26) = 1. Candidates are {1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25}
- Therefore, the key space has  $12 \cdot 26 = 312$  candidates.
- Attack types:

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- *Ciphertext only:* exhaustive search or frequency analysis

# Letter Frequency Analysis

Consider the ciphertext
 FMNVEDKAPHFERBNDKRX

RSREFMORUDSDKDVSHVU FEDKAPRKDLYEVLRHHRH

• Letter frequency of the ciphertext:

	-
Letter	# of Occurrences
R	8
D	6
E	5
Н	5
K	5
V	4
F	4

# Letter Frequency Analysis

- Better Solution: correlation
  - Enumerate 312 possible keys, ex. (3,2)
  - $\text{Let } A_0 = [.082, .015, .028, .043, .127, .022, .020, .061, .070, \\.002, .008, .040, .024, .067, .075, .019, .001, .060, \\.063, .091, 028, .010, .023, .001, .020, .001]$
  - Let the i-th key be (3,2), which maps plaintexts [0, 1, 2, 3, 4 ..., 25] to ciphertexts [2, 5, 8, 11, 14, 17, 20, 23, ...]
  - Calculate a vector  $A_i$  with the k-th element being  $A_0(E_{3,2}(k))$ , ex.  $A_i = [A_0(2), A_0(5), A_0(8), A_0(11), A_0(14), A_0(17), A_0(20), A_0(23), A_0(0), ...]$

– Perform correlation  $A \cdot A_i$  and find the maximum

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# Letter Frequency Analysis

- Make a guess: choose two potential candidate letters e.g. 1st guess  $R \rightarrow e$  and  $D \rightarrow t$
- Try to show the guess make sense by solving (α, β) from E<sub>k</sub>(x) = Y = α · x + β mod 26 e.g. 4 α + β =17 mod 26 and 19 α + β =5 mod 26 ⇒ α = 6, β =19, which is illegal since gcd(6,26)>1
- 2nd guess:  $R \rightarrow e$  and  $E \rightarrow t \dots \Rightarrow \alpha = 13$ , still illegal
- 3rd guess:  $\mathbb{R} \to e$  and  $\mathbb{H} \to t$  .....  $\Rightarrow \alpha = 3, \beta = 5$ i.e.  $E_k(x) = 3 \cdot x + 5 \mod 26$  $D_k(x) = 9 \cdot x - 19 \mod 26$

# Affine Cipher

#### • Attack types:

- Known plaintext: two letters in the plaintext and corresponding ciphertext letters would suffice to find the key.
  - Ex. plaintext 'if'=(8, 5) and ciphertext 'PQ'=(15, 16)  $8 \cdot \alpha + \beta \equiv 15 \mod 26$  $5 \cdot \alpha + \beta \equiv 16 \mod 26 \implies \alpha = 17 \text{ and } \beta = 9$

What happens if we have only one letter of known plaintext?

still have great reduction in candidates

# Affine Cipher

#### • Attack types:

- *Chosen plaintext:* Choose a and b as the plaintext. The first character of the ciphertext will be equal to  $0 \cdot \alpha + \beta = \beta$  and the second will be  $\alpha + \beta$ .
- *Chosen ciphertext*: Choose A and B as the ciphertext. The first character of the plaintext will be equal to  $0 \cdot \alpha' + \beta' = \beta'$  and the second will be  $\alpha' + \beta'$ ,  $\alpha = (\alpha')^{-1}$  and  $\beta = -\alpha \cdot \beta'$

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# Matlab Example

- a = 3, b = 5, ap = 9, bp = -19;
- plain = 'matlabaffinecipherencryptionexample';
- cipher = affinecrypt(plain, a, b)
- recovered = affinecrypt(cipher, ap, bp)

# Substitution Ciphers

- Each letter in the alphabet is replaced (substituted) by another letter. More precisely, a permutation of the alphabet is chosen and applied to the plaintext.
- <u>Shift ciphers and affine ciphers are special cases of</u> substitution ciphers.
- Since ciphertext preserves the statistic of the language used in the plaintext, the "frequency analysis" is an effective way of breaking substitution ciphers with only ciphertext.
- The Adventure of the Dancing Men by Arthur Conan Doyle http://www.sherlockian.net/canon/stories/danc.html

# \*\*\*\*

#### Vigenère Cipher

- Algorithm: Let  $P = C = Z_{26}$  and  $x \in P$ ,  $Y \in C$ - *Encryption*:  $Y = E_{\iota}(x) \equiv x + k_{i} \pmod{26}$ 
  - The key  $k = (k_1, k_2, k_3, ..., k_n), k_i \in Z_{26}$ , neither the key or the length n is known to adversary
  - Decryption:  $x = D_{i}(Y) \equiv Y k_{i} \pmod{26}$
- eX. key='danger' plaintext: hellothisisa keys: dangerdanger
- Attacks: ciphertext: KEYRSKKIFOWR
  - Ciphertext Only:
    - Finding the key length
    - Finding the key

#### Vigenère Cipher

- Finding the key length:
  - Friedman's Test uses Index of Coincidence: Let  $I_c(x)$  be the probability that two random elements of the n-letter string x are identical
  - Let f<sub>0</sub>, f<sub>1</sub>, ..., f<sub>25</sub> be the number of occurrence of A, B, ...Z, respectively in the n-letter string x

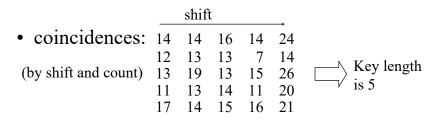
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#### Vigenère Cipher

 find the coincidences in the ciphertext 'vvhqwvvrhmusgjgthkihtssejchlsfcbgvwcrlryqtfsvgahwkcuhwauglq' 'hnslrljshbltspisprdxljsveeghlqwkasskuwepwqtwvspgoelkcqyfnsv'

'wljsniqkgnrgybwlwgoviokhkazkqkxzgyhcecmeiujoqkwfwvefqhkijrc' 'lrlkbienqfrjljsdhgrhlsfqtwlauqrhwdmwlgusgikkflryvcwvspgpmlk' 'assjvoqxeggveyggzmljcxxljsvpaivwikvrdrygfrjljslveggveyggeia' 'puuisfpbtgnwwmuczrvtwglrwugumnczvile'



# Vigenère Cipher • The letter frequency of English is - $A_0 = [.082 .015 .028 .043 .127 .022 .020 .061 .070 .002 .008 .040 .024 .067 .075 .019 .001 .060 .063 .091 .028 .010 .023 .001 .020 .001] • The expected value of <math>I_c(x)$ is - for English Text: $I_c(x) = A_0 \cdot A_0 = (.082)^2 + (.015)^2 + ... = 0.666$ - for Random String: $I_c(x) = 26 \cdot (1/26)^2 = 0.038$ - for shifted English Text(the first letter shifted by $k_i$ and the second letter shifted by $k_j$ ): 1 2 3 4 5 6 7 8 9 10 11 12 13 (a) $A_0 = .039 .034 .034 .034 .038 .045 .039 .042$ (b) $A_0 = .039 .034 .034 .034 .038 .045 .039 .042$

#### Vigenère Cipher

#### • Finding the Key:

 To find the first element of the key, count the frequencies of the letters in the 1st, 6th, 11th ... positions of the ciphertext

V = (0,0,7,1,1,2,9,0,1,8,8,0,0,3,0,4,5,2,0,3,6,5,1,0,1,0)

- Divide by number of letters counted, 67
   W = (0, 0, .1045,.0149,.0149,.0299,...,.0149,0)

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#### Vigenère Cipher

- Known plaintext:
  - if enough (plaintext, ciphertext) pairs are known
     k<sub>i</sub> = Y x
- Chosen plaintext:
  - choose plaintext aaaaa...

 $k_i = Y$ 

- Chosen ciphertext:
  - choose ciphertext AAAAA...

 $k_i = -x$ 

# Matlab Example

- Encrypt/decrypt
  - key = 'vigenere';
  - key\_i = text2int(key);
  - plain = 'matlabaffinecipherencryptionexample';
  - cipher=vigenere(plain, key\_i)
  - recovered=vigenere(cipher, -key\_i)

# Matlab Example

- Ciphertext only attack:
  - ciphertexts
  - for i=1:25,
  - a(i) = coinc(vvhq, i);

finding key length

finding first key

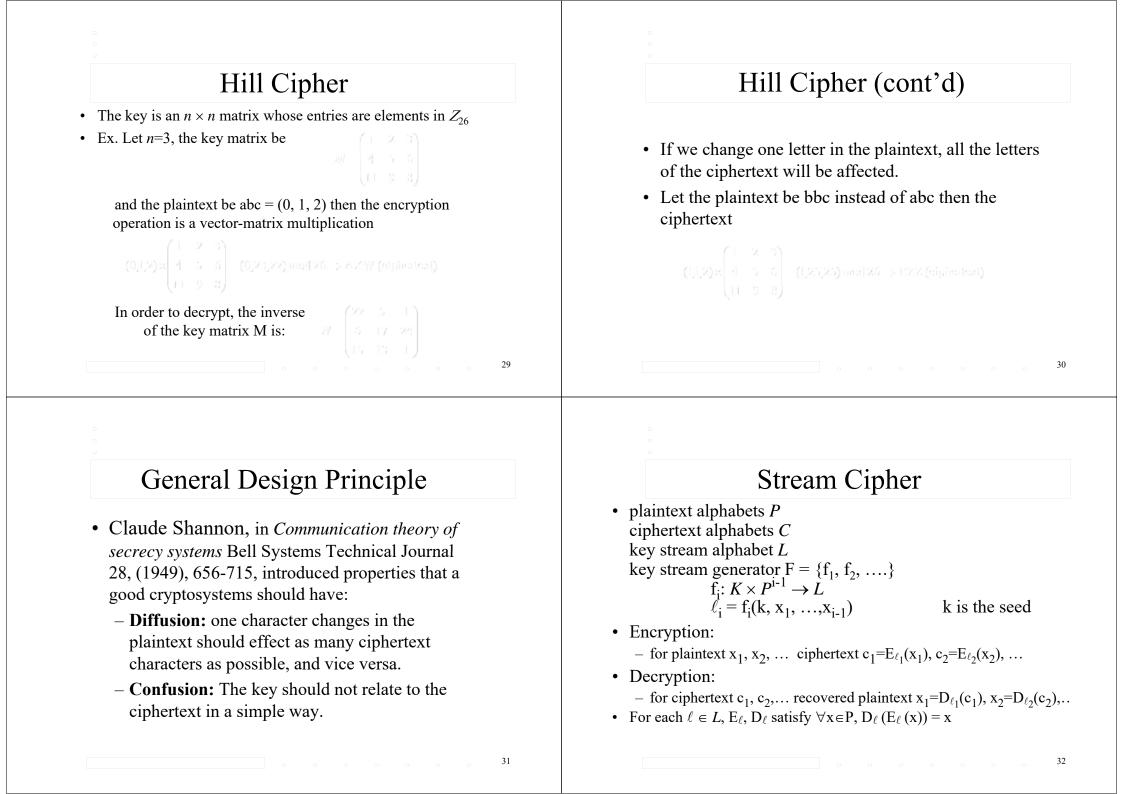
- end
- first = choose(vvhq, 5, 1)
- V = frequency(first)
- W = V / length(first)
- corr(W)

# **Block** Ciphers

- In the substitution ciphers, changing one letter in the plaintext changes exactly one letter in the ciphertext.
- This greatly facilitates finding the key using frequency analysis.
- Block ciphers prevent this by encrypting a block of letters simultaneously.
- Many of the modern (symmetric) cryptosystems are block ciphers. DES operates on 64 bits of blocks while AES uses 128 bits of blocks (optionally 192 and 256 bits blocks).

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## Autokey cipher

- Key stream generator: l<sub>i</sub> = x<sub>i-1</sub>, l<sub>1</sub> = k, k is an initial seed Encryption: E<sub>l</sub> (x) = x + l mod 26 Decryption: D<sub>l</sub> (y) = y - l mod 26
- Ex: k = 8, plaintext: 'rendezvouz'

nlaintayt.	r	e	n	d	e	Ζ	$\mathbf{V}$	0	u	S	
plaintext.	17	4	13	3	4	25	21	14	20	18	
plaintext: { keys:	8	17	<b>`</b> 4	13	3	4	25	21	14	20	18
ciphertext:{	25	21	17	16	7	3	20	22	8	12	
cipitertext.	Ζ	V	R	Q	Η	D	U	J	Ι	Μ	
keys:	8	,17	, 4	,13	3	4	25	21	14	20	18
keys: plaintext:	17	4	13	3	4	25	21	14	20	18	

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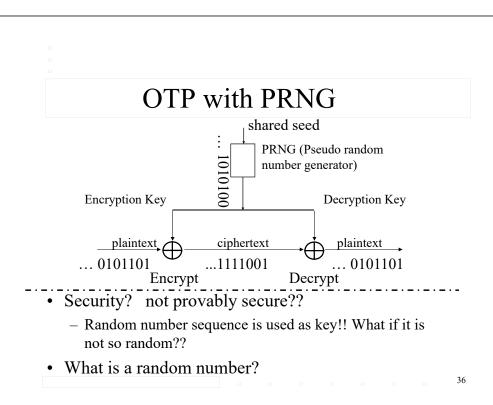
# Stream Cipher

- Block ciphers are special cases of stream ciphers where the key stream is constant.
- A stream cipher is synchronous if the key stream is independent of the plaintext.
  - Both sender and receiver must be synchronized.
  - Resynchronization can be needed.
  - No error propagation (if the deciphered plaintext is incorrect).
  - $-\,$  Active attacks can easily be detected.
- A stream cipher is periodic with period d if  $\ell_{i+d} = \ell_i$ , for all  $i \ge 1$ .

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# Stream Cipher

- The Vigenère cipher with keyword length m is a periodic stream cipher with period m.
- Stream ciphers are often described in binary 0, 1 alphabets. ex. one-time pad
- Perfectly Secure: One-time pad
- Examples of practical stream ciphers
  - Autokey Cipher
  - One-time pad with Pseudo-random Bit Generation
  - Linear Feedback Shift Register (LFSR)
  - $-\,$  DES in Counter Mode or CFB Mode
  - Feistel Cipher



# Randomness

- Randomness? ex. flipping a fair coin, thermal noise
  - Uniformly distributed string sequences
  - a string s is Komogorov-random if its length equals the length of the shortest program producing s
     ex. 010101010101010101
- Statistical approach: pass some statistical tests: ex. 0/1 bits appear equally, number of 0/1 bits are equal, any two bits are uncorrelated, Maurer's Universal Test, Chi-Square Test, Kolmogorov-Smirnov Test ...
  - Computational approach:

random

pseudo random, PRNG

- indistinguishable from any uniformly distributed sequences
- unpredictable by any poly-time algorithm (the probability to predict the next bit is no better than 1/2)

# Pseudorandom Number Generator

- Existence? one way function assumption
- Poor implementation for cryptographic usage:
  - linear congruential generator rand() in the standard C/UNIX library
    - $x_n = a x_{n-1} + b \mod m$ ,  $x_0$  is the initial seed
    - a, b, m can be discovered from the  $x_n$  sequence
    - therefore *x<sub>n</sub>* is completely predictable (key is know to everybody!!)
    - any polynomial congruential generator is cryptographically insecure
    - can be used only for the purpose of statistical experiments

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# Pseudorandom Number Generator

- Fairly good implementation for cryptographic purpose:
  - Method 1: based on one-way function candidates (DES, SHA..)
    - one-way function f: y = f(x), given y, it's hard to compute x
      - $x_i = f(s+j), j=1,2,3,...$  s is the seed
      - let the random bit sequence  $b_i$  be the LSB of  $x_i$ ,
    - PRNG in the OpenSSL toolkit is based on SHA
  - Method 2: Blum-Blum-Shub (BBS, 1984)
    - $p \equiv 3 \pmod{4}$ ,  $q \equiv 3 \pmod{4}$ ,  $n \equiv p \cdot q$ , seed k
    - x<sub>0</sub> ≡ k<sup>2</sup> (mod n), x<sub>j</sub> ≡ x<sub>j-1</sub><sup>2</sup> (mod n),
       let the random bit sequence b<sub>j</sub> be the LSB of x<sub>j</sub>

# BBS example

• Let  $p = 24672462467892469787 \quad q = 396736894567834589803$  n = 9788476140853110794168855217413715781961take k = 873245647888478349013

 $\begin{aligned} x_0 &\equiv k^2 \pmod{n} \equiv 8845298710478780097089917746010122863172 \\ x_1 &\equiv x_0^2 \pmod{n} \equiv 7118894281131329522745962455498123822408 \\ x_2 &\equiv x_1^2 \pmod{n} \equiv 3145174608888893164151380152060704518227 \end{aligned}$ 

 $b_1 = 0$ 

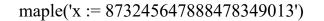
. . . .

$$p_2 = 1, \dots$$

 slow for practical application, take k (≤ log<sub>2</sub>log<sub>2</sub>n) LSB bits of x<sub>j</sub>

# Maple example in Matlab

maple('p := 24672462467892469787') maple('q := 396736894567834589803') maple('n := p\*q')



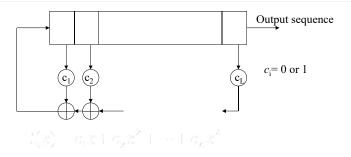
maple('x0 :=  $x\&^2 \mod n'$ ) maple('x1 :=  $x0\&^2 \mod n'$ ) maple('x2 :=  $x1\&^2 \mod n'$ )

...



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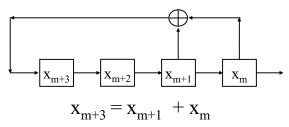




- If C(x) is primitive, LFSR is called *maximum-length LFSR*, and the output sequence is called *m-sequence* and its period is  $T = 2^{L}-1$ .
- *m-sequences* have good statistical properties.
- However, they are predictable.

Linear Feedback Shift Register (LFSR)

• Hardware-oriented implementation: sacrifice security to obtain encryption speed



• in general:

 $x_{n+m} = c_0 x_n + c_1 x_{n+1} + ... + c_{m-1} x_{n+m-1} \pmod{2}$ with initial values  $x_1, x_2, ..., x_m$ 

## Linear Feedback Shift Register (LFSR)

- For a length m linear recurrence relation, the period of the sequence is at most 2<sup>m</sup>-1.
  - Any m consecutive terms of the sequence determine the complete sequence. As soon as there are more than 2<sup>m</sup>-1 terms, some string of length m must occur twice.



0-th m-bit group



(2<sup>m</sup>-2)-th m-bit group

- ex.  $x_{n+31} \equiv x_n + x_{n+3}$ , with any **nonzero initial vector**, will produce a sequence that has period  $2^{31}$ -1

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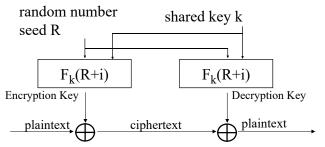
# Linear Feedback Shift Register (LFSR)

- Given a segment 011010111100 of a LFSR sequence, it is possible to deduce the length of the recurrence and the coefficients. (If you find a segment of 2m-bit plaintext and the corresponding ciphertext, you discover the corresponding segment of the key sequence.)
- The general solution: solve coefficients c<sub>i</sub> from

# DES in Counter Mode

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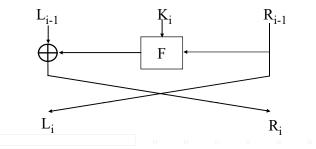
- pseudo one-time pad
- has better security properties than CBC, CFB, OFB encryption modes

# Linear Feedback Shift Register (LFSR)

- Computation in GF(2<sup>n</sup>) can be quickly implemented in hardware with linear-feedback shift registers.
- Computation in GF(2<sup>n</sup>) (eg. exponentiation and discrete log) is often quicker than computation over GF(p).
  - E. R. Berlekamp, Algebraic Coding Theory, Aegean Park press 1984
  - T. Beth et. al, "Architectures for Exponentiation in GF(2<sup>n</sup>)," Crypto 86

## Feistel Cipher

- Horst Feistel, 1973 IBM LUCIFER
- a common block encryption structure used in many symmetric encryption schemes that maximize the effects of Shannon's "Confusion" and "Diffusion"



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# Enigma

- German Enigma cipher machine in World War II. The Enigma had been broken by the Allies in World War II. The capture of the German U-505 submarine in David Kahn's book.
- U-571, 2000 movie; Enigma, 2002 movie
- see John J. G. Savard, A Cryptographic Compendium

- $-\ http://home.ecn.ab.ca/{\sim} jsavard/crypto/entry.htm$
- Codes throughout history
  - $-\ http://codebreaker.dids.com/fhistory.htm$