

ZeroJudge b537. 分數運算-1

<https://zerojudge.tw/ShowProblem?problemid=b537>

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113/10/31

題目

題目

- $f(1)=1$

題目

- $f(1)=1$
- $f(k)=1+f(k/2)$, k is even

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- $f(1)=1$
 - $f(k)=1+f(k/2)$, k is even
 - $f(k)=1/f(k-1)$, k is odd

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$$f(k)=a/b$$

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$$f(k)=a/b \Rightarrow$$

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$$f(k)=a/b \quad \Rightarrow \quad k=?$$

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$$f(k)=a/b \quad \Rightarrow \quad k=?$$

e.g.

$$f(6) = 1 + f(3)$$

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$$f(k)=a/b \Rightarrow k=?$$

e.g.

$$f(6) = 1 + f(3)$$

$$f(8) = 1 + f(4)$$

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$$f(k)=a/b \quad \Rightarrow \quad k=?$$

e.g.

$$f(6) = 1 + f(3) \quad f(9) = 1 / f(8)$$

$$f(8) = 1 + f(4)$$

題目

- $f(1)=1$
- $f(k)=1+f(k/2)$, k is even
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$$f(k)=a/b \quad \Rightarrow \quad k=?$$

e.g.

$$f(6) = 1 + f(3)$$

$$f(9) = 1 / f(8)$$

$$f(8) = 1 + f(4)$$

$$f(11) = 1 / f(10)$$

題目

- $f(1)=1$
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$$f(k)=a/b \Rightarrow k=?$$

範例輸入

1 1

2 1

1 2

3 1

1 3

3 2

2 3

4 1

1 4

4 3

30 11

e.g.

$$f(6) = 1 + f(3)$$

$$f(9) = 1 / f(8)$$

$$f(8) = 1 + f(4)$$

$$f(11) = 1 / f(10)$$

題目

- $f(1)=1$
- $f(k)=1+f(k/2)$, k is even
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$$f(k)=a/b \Rightarrow k=?$$

範例輸入

1 1

2 1

1 2

3 1

1 3

3 2

2 3

4 1

1 4

4 3

30 11

範例輸出

1

2

3

4

5

6

7

8

9

10

236

e.g.

$$f(6) = 1 + f(3)$$

$$f(9) = 1 / f(8)$$

$$f(8) = 1 + f(4)$$

$$f(11) = 1 / f(10)$$

從範例測資開始分析

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$$\frac{30}{11}$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11}$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11}$$

$f(k)$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11}$$

$f(2k)$ $f(k)$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(k)$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(2k)$ $f(k)$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11} \quad \left| \quad \frac{8}{11}\right.$$

$f(4k)$ $f(2k)$ $f(k)$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11} \quad \left| \quad \frac{8}{11} = \frac{1}{\frac{11}{8}}\right.$$

$f(4k)$ $f(2k)$ $f(k)$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$

$$\frac{8}{11} = \frac{1}{\frac{11}{8} f(k)}$$

從範例測資開始分析

$$\begin{array}{c} \frac{30}{11} \\ f(4k) \end{array} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11} \quad \left| \quad \begin{array}{c} \frac{8}{11} \\ f(k+1) \end{array} = \frac{1}{\frac{11}{8} f(k)}$$

從範例測資開始分析

$$\begin{array}{ccc|cc|c} \frac{30}{11} & = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11} & & \frac{8}{11} & = \frac{1}{11} & & \frac{11}{8} \\ f(4k) & f(2k) & f(k) & f(k+1) & \frac{11}{8} f(k) & & \end{array}$$

從範例測資開始分析

$$\begin{array}{ccc|c|c} \frac{30}{11} & = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11} & & \frac{8}{11} = \frac{1}{11} & \frac{11}{8} = 1 + \frac{3}{8} \\ f(4k) & f(2k) & f(k) & f(k+1) & \frac{11}{8} f(k) \end{array}$$

從範例測資開始分析

$$\begin{array}{ccc} \frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11} & \frac{8}{11} = \frac{1}{11} & \frac{11}{8} = 1 + \frac{3}{8} \\ f(4k) & f(k+1) & f(k) \end{array}$$

從範例測資開始分析

$$\begin{array}{ccc|ccc|ccc} \frac{30}{11} & = 1 + \frac{19}{11} & = 1 + 1 + \frac{8}{11} & & \frac{8}{11} & = \frac{1}{11} & & \frac{11}{8} & = 1 + \frac{3}{8} \\ f(4k) & & f(2k) & & f(k+1) & & f(k) & & f(2k) & & f(k) \end{array}$$

從範例測資開始分析

$$\frac{\textcolor{blue}{30}}{\textcolor{blue}{11}} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k) \quad f(2k) \quad f(k)$

$$\frac{8}{11} = \frac{1}{\frac{11}{8}} f(k)$$

$f(k+1) \quad \frac{11}{8} f(k)$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k) \quad f(k)$

$$\frac{3}{8}$$

從範例測資開始分析

$$\frac{\overset{30}{\underset{11}{f(4k)}}}{f(4k)} = 1 + \frac{19}{f(2k)} = 1 + 1 + \frac{8}{f(k)}$$

$$\frac{8}{f(k+1)} = \frac{1}{\frac{11}{8} f(k)}$$

$$\frac{11}{8} = 1 + \frac{3}{f(2k)} = 1 + \frac{3}{f(k)}$$

$$\frac{3}{8} = \frac{1}{\frac{8}{3}}$$

從範例測資開始分析

$$\frac{\overset{30}{\underset{11}{f(4k)}}}{f(4k)} = 1 + \frac{19}{f(2k)} = 1 + 1 + \frac{8}{f(k)}$$

$$\frac{8}{f(k+1)} = \frac{1}{\frac{11}{8} f(k)}$$

$$\frac{11}{8} = 1 + \frac{3}{f(2k)} = 1 + \frac{3}{f(k)}$$

$$\frac{3}{8} = \frac{1}{\frac{8}{3} f(k)}$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{8}{11} = \frac{1}{11}$$

$$f(k+1) \quad \frac{1}{8} f(k)$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$f(2k) \quad f(k)$$

$$\frac{3}{8} = \frac{1}{8}$$

$$f(k+1) \quad \frac{1}{3} f(k)$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k) \quad f(2k) \quad f(k)$

$$\frac{8}{11} = \frac{1}{11} \cdot 8$$

$f(k+1) \quad \frac{1}{11} f(k)$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k) \quad f(k)$

$$\frac{3}{8} = \frac{1}{8} \cdot 3$$

$f(k+1) \quad \frac{1}{8} f(k)$

$$\frac{8}{3}$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$

$$\frac{8}{11} = \frac{1}{11} \cdot \frac{8}{1}$$

$f(k+1)$ $\frac{1}{8} f(k)$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k)$ $f(k)$

$$\frac{3}{8} = \frac{1}{8} \cdot \frac{3}{1}$$

$f(k+1)$ $\frac{1}{3} f(k)$

$$\frac{8}{3} = 1 + \frac{5}{3}$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k) \quad f(2k) \quad f(k)$

$$\frac{8}{11} = \frac{1}{11}$$

$f(k+1) \quad \frac{11}{8} f(k)$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k) \quad f(k)$

$$\frac{3}{8} = \frac{1}{8}$$

$f(k+1) \quad \frac{8}{3} f(k)$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

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$f(4k)$ $f(2k)$ $f(k)$

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$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

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$f(2k)$ $f(k)$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$

$$\frac{8}{11} = \frac{1}{11} \cdot \frac{11}{8} f(k)$$

$f(k+1)$ $\frac{11}{8} f(k)$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k)$ $f(k)$

$$\frac{3}{8} = \frac{1}{8} \cdot \frac{8}{3} f(k)$$

$f(k+1)$ $\frac{8}{3} f(k)$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$f(4k)$ $f(2k)$ $f(k)$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

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$$f(2k) \quad f(k)$$

$$\frac{3}{8} = \frac{1}{8}$$

$$f(k+1) \quad \frac{1}{3} f(k)$$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3}$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{8}{11} = \frac{1}{11}$$

$$f(k+1) \quad \frac{1}{8} f(k)$$

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$$f(2k) \quad f(k)$$

$$\frac{3}{8} = \frac{1}{8}$$

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$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3} = \frac{1}{3}$$

$$\frac{1}{2}$$

從範例測資開始分析

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$$f(k+1) \quad \frac{1}{3} f(k)$$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3} = \frac{1}{3}$$

$$\frac{1}{2} f(k)$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

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$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3} = \frac{1}{3}$$

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從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

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$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$$f(4k) \quad f(2k) \quad f(k)$$

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從範例測資開始分析

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$$f(k)$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{8}{11} = \frac{1}{11}$$

$$f(k+1) \quad \frac{1}{8} f(k)$$

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$$f(2k) \quad f(k)$$

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$$f(4k) \quad f(2k) \quad f(k)$$

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$$f(k+1) \quad \frac{1}{2} f(k)$$

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$$f(2k) \quad f(k)$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{8}{11} = \frac{1}{11}$$

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$$\frac{2}{3} = \frac{1}{3}$$

$$f(k+1) \quad \frac{1}{2} f(k)$$

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$$f(2k) \quad f(k)$$

$$\frac{1}{2}$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

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$$f(2k) \quad f(k)$$

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$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3} = \frac{1}{3}$$

$$f(k+1) \quad \frac{1}{2} f(k)$$

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$$f(2k) \quad f(k)$$

$$\frac{1}{2} = \frac{1}{2}$$

$$1$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{8}{11} = \frac{1}{11}$$

$$f(k+1) \quad \frac{1}{8} f(k)$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$f(2k) \quad f(k)$$

$$\frac{3}{8} = \frac{1}{8}$$

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$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3} = \frac{1}{3}$$

$$f(k+1) \quad \frac{1}{2} f(k)$$

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$$f(2k) \quad f(k)$$

$$\frac{1}{2} = \frac{1}{2}$$

$$f(k)$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{8}{11} = \frac{1}{11}$$

$$f(k+1) \quad \frac{1}{8} f(k)$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$f(2k) \quad f(k)$$

$$\frac{3}{8} = \frac{1}{8}$$

$$f(k+1) \quad \frac{1}{3} f(k)$$

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$$f(4k) \quad f(2k) \quad f(k)$$

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$$f(k+1) \quad \frac{1}{2} f(k)$$

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$$f(2k) \quad f(k)$$

$$\frac{1}{2} = \frac{1}{2}$$

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從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

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$$f(2k) \quad f(k)$$

$$\frac{3}{8} = \frac{1}{8}$$

$$f(k+1) \quad \frac{1}{3} f(k)$$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3} = \frac{1}{3}$$

$$f(k+1) \quad \frac{1}{2} f(k)$$

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$$f(2k) \quad f(k)$$

$$\frac{1}{2} = \frac{1}{2}$$

$$f(k+1) \quad \frac{1}{1} f(k)$$

$$\frac{2}{1}$$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{8}{11} = \frac{1}{11}$$

$$f(k+1) \quad \frac{1}{8} f(k)$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$f(2k) \quad f(k)$$

$$\frac{3}{8} = \frac{1}{8}$$

$$f(k+1) \quad \frac{1}{3} f(k)$$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3} = \frac{1}{3}$$

$$f(k+1) \quad \frac{1}{2} f(k)$$

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$$f(2k) \quad f(k)$$

$$\frac{1}{2} = \frac{1}{2}$$

$$f(k+1) \quad \frac{1}{1} f(k)$$

$$\frac{2}{1} = 1 + 1$$

從範例測資開始分析

$$\frac{\textcolor{blue}{30}}{\textcolor{blue}{11}} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{8}{11} = \frac{1}{\frac{11}{8}} f(k)$$

$$f(k+1) \quad \frac{11}{8} f(k)$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$f(2k) \quad f(k)$$

$$\frac{3}{8} = \frac{1}{\frac{8}{3}} f(k)$$

$$f(k+1) \quad \frac{8}{3} f(k)$$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3} = \frac{1}{\frac{3}{2}} f(k)$$

$$f(k+1) \quad \frac{3}{2} f(k)$$

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$$f(2k) \quad f(k)$$

$$\frac{1}{2} = \frac{1}{\frac{2}{1}} f(k)$$

$$f(k+1) \quad \frac{2}{1} f(k)$$

$$\frac{2}{1} = 1 + 1$$

$$f(\textcolor{green}{1})$$

從範例測資開始分析

$$\frac{\textcolor{blue}{30}}{\textcolor{blue}{11}} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{8}{11} = \frac{1}{\frac{11}{8}} f(k)$$

$$f(k+1) \quad \frac{11}{8} f(k)$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$f(2k) \quad f(k)$$

$$\frac{3}{8} = \frac{1}{\frac{8}{3}} f(k)$$

$$f(k+1) \quad \frac{8}{3} f(k)$$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3} = \frac{1}{\frac{3}{2}} f(k)$$

$$f(k+1) \quad \frac{3}{2} f(k)$$

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$$f(2k) \quad f(k)$$

$$\frac{1}{2} = \frac{1}{\frac{2}{1}} f(k)$$

$$f(k+1) \quad \frac{2}{1} f(k)$$

$$\frac{2}{1} = 1 + 1$$

$$f(\textcolor{blue}{2}) \quad f(\textcolor{green}{1})$$

從範例測資開始分析

$$\frac{\textcolor{blue}{30}}{\textcolor{blue}{11}} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{8}{11} = \frac{1}{\frac{11}{8}} f(k)$$

$$f(k+1) \quad \frac{11}{8} f(k)$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$f(2k) \quad f(k)$$

$$\frac{3}{8} = \frac{1}{\frac{8}{3}} f(k)$$

$$f(k+1) \quad \frac{8}{3} f(k)$$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3} = \frac{1}{\frac{3}{2}} f(k)$$

$$f(k+1) \quad \frac{3}{2} f(k)$$

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$$f(2k) \quad f(k)$$

$$\frac{1}{2} = \frac{1}{\frac{2}{1}} f(k)$$

$$f(k+1) \quad \frac{2}{1} f(k)$$

2

$$\frac{2}{1} = 1 + 1$$

$$f(\textcolor{blue}{2}) \quad f(\textcolor{green}{1})$$

從範例測資開始分析

$$\frac{\textcolor{blue}{30}}{\textcolor{blue}{11}} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{8}{11} = \frac{1}{\frac{11}{8}} f(k)$$

$$f(k+1) \quad \frac{11}{8} f(k)$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$f(2k) \quad f(k)$$

$$\frac{3}{8} = \frac{1}{\frac{8}{3}} f(k)$$

$$f(k+1) \quad \frac{8}{3} f(k)$$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3} = \frac{1}{\frac{3}{2}} f(k)$$

$$f(k+1) \quad \frac{3}{2} f(k)$$

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$$f(2k) \quad f(k)$$

$$\frac{1}{2} = \frac{1}{\frac{2}{1}} f(k)$$

$$f(k+1) \quad \frac{2}{1} f(k)$$

3 **2**

$$\frac{2}{1} = 1 + 1$$

$$f(\textcolor{blue}{2}) \quad f(\textcolor{green}{1})$$

從範例測資開始分析

$$\frac{\mathbf{30}}{\mathbf{11}} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{8}{11} = \frac{1}{\frac{11}{8}} f(k)$$

$$f(k+1) \quad \frac{11}{8} f(k)$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$f(2k) \quad f(k)$$

$$\frac{3}{8} = \frac{1}{\frac{8}{3}} f(k)$$

$$f(k+1) \quad \frac{8}{3} f(k)$$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3} = \frac{1}{\frac{3}{2}} f(k)$$

$$f(k+1) \quad \frac{3}{2} f(k)$$

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$$f(2k) \quad f(k)$$

3

$$\frac{1}{2} = \frac{1}{\frac{2}{1}} f(k)$$

$$f(k+1) \quad \frac{2}{1} f(k)$$

3 **2**

$$\frac{2}{1} = 1 + 1$$

$$f(\mathbf{2}) \quad f(\mathbf{1})$$

從範例測資開始分析

$$\frac{\textcolor{blue}{30}}{\textcolor{blue}{11}} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{8}{11} = \frac{1}{\frac{11}{8}} f(k)$$

$$f(k+1) \quad \frac{11}{8} f(k)$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$f(2k) \quad f(k)$$

$$\frac{3}{8} = \frac{1}{\frac{8}{3}} f(k)$$

$$f(k+1) \quad \frac{8}{3} f(k)$$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3} = \frac{1}{\frac{3}{2}} f(k)$$

$$f(k+1) \quad \frac{3}{2} f(k)$$

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$$f(2k) \quad f(k)$$

$$\textcolor{blue}{6} \quad \textcolor{blue}{3}$$

$$\frac{1}{2} = \frac{1}{\frac{2}{1}} f(k)$$

$$f(k+1) \quad \frac{2}{1} f(k)$$

$$\textcolor{blue}{3} \quad \textcolor{blue}{2}$$

$$\frac{2}{1} = 1 + 1$$

$$f(\textcolor{blue}{2}) \quad f(\textcolor{green}{1})$$

從範例測資開始分析

$$\frac{\textcolor{blue}{30}}{\textcolor{blue}{11}} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{8}{11} = \frac{1}{\frac{11}{8}} f(k)$$

$$f(k+1) \quad \frac{11}{8} f(k)$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$f(2k) \quad f(k)$$

$$\frac{3}{8} = \frac{1}{\frac{8}{3}} f(k)$$

$$f(k+1) \quad \frac{8}{3} f(k)$$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$$f(4k) \quad f(2k) \quad f(k)$$

$$\frac{2}{3} = \frac{1}{\frac{3}{2}} f(k)$$

$$f(k+1) \quad \frac{3}{2} f(k)$$

6

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$$f(2k) \quad f(k)$$

6 **3**

$$\frac{1}{2} = \frac{1}{\frac{2}{1}} f(k)$$

$$f(k+1) \quad \frac{2}{1} f(k)$$

3 **2**

$$\frac{2}{1} = 1 + 1$$

$$f(2) \quad f(\textcolor{green}{1})$$

從範例測資開始分析

$$\frac{\mathbf{30}}{\mathbf{11}} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$

$$\frac{8}{11} = \frac{1}{\frac{11}{8}} f(k)$$

$f(k+1)$ $\frac{11}{8} f(k)$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k)$ $f(k)$

$$\frac{3}{8} = \frac{1}{\frac{8}{3}} f(k)$$

$f(k+1)$ $\frac{8}{3} f(k)$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$f(4k)$ $f(2k)$ $f(k)$

$$\frac{2}{3} = \frac{1}{\frac{3}{2}} f(k)$$

$f(k+1)$ $\frac{3}{2} f(k)$

7 **6**

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$f(2k)$ $f(k)$

6 **3**

$$\frac{1}{2} = \frac{1}{\frac{2}{1}} f(k)$$

$f(k+1)$ $\frac{2}{1} f(k)$

3 **2**

$$\frac{2}{1} = 1 + 1$$

$f(2)$ $f(\mathbf{1})$

2

從範例測資開始分析

$$\frac{\mathbf{30}}{\mathbf{11}} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$

$$\frac{8}{11} = \frac{1}{\frac{11}{8}} f(k)$$

$f(k+1)$ $\frac{11}{8} f(k)$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k)$ $f(k)$

$$\frac{3}{8} = \frac{1}{\frac{8}{3}} f(k)$$

$f(k+1)$ $\frac{8}{3} f(k)$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$f(4k)$ $f(2k)$ $f(k)$

7

$$\frac{2}{3} = \frac{1}{\frac{3}{2}} f(k)$$

$f(k+1)$ $\frac{3}{2} f(k)$

7 **6**

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$f(2k)$ $f(k)$

6 **3**

$$\frac{1}{2} = \frac{1}{2} f(k)$$

$f(k+1)$ $\frac{2}{1} f(k)$

3 **2**

$$\frac{2}{1} = 1 + 1$$

$f(\mathbf{2})$ $f(\mathbf{1})$

從範例測資開始分析

$$\frac{\mathbf{30}}{\mathbf{11}} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$

$$\frac{8}{11} = \frac{1}{\frac{11}{8}} f(k)$$

$f(k+1)$ $\frac{11}{8} f(k)$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k)$ $f(k)$

$$\frac{3}{8} = \frac{1}{\frac{8}{3}} f(k)$$

$f(k+1)$ $\frac{8}{3} f(k)$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$f(4k)$ $f(2k)$ $f(k)$

14 **7**

$$\frac{2}{3} = \frac{1}{\frac{3}{2}} f(k)$$

$f(k+1)$ $\frac{3}{2} f(k)$

7 **6**

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$f(2k)$ $f(k)$

6 **3**

$$\frac{1}{2} = \frac{1}{2} f(k)$$

$f(k+1)$ $\frac{2}{1} f(k)$

3 **2**

$$\frac{2}{1} = 1 + 1$$

$f(\mathbf{2})$ $f(\mathbf{1})$

從範例測資開始分析

$$\frac{\mathbf{30}}{\mathbf{11}} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$

$$\frac{8}{11} = \frac{1}{\frac{11}{8}} f(k)$$

$f(k+1)$ $\frac{11}{8} f(k)$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k)$ $f(k)$

$$\frac{3}{8} = \frac{1}{\frac{8}{3}} f(k)$$

$f(k+1)$ $\frac{8}{3} f(k)$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$f(4k)$ $f(2k)$ $f(k)$

28 **14** **7**

$$\frac{2}{3} = \frac{1}{\frac{3}{2}} f(k)$$

$f(k+1)$ $\frac{3}{2} f(k)$

7 **6**

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$f(2k)$ $f(k)$

6 **3**

$$\frac{1}{2} = \frac{1}{2} f(k)$$

$f(k+1)$ $\frac{2}{1} f(k)$

3 **2**

$$\frac{2}{1} = 1 + 1$$

$f(\mathbf{2})$ $f(\mathbf{1})$

從範例測資開始分析

$$\frac{\mathbf{30}}{\mathbf{11}} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$

$$\frac{8}{11} = \frac{1}{\frac{11}{8}} f(k)$$

$f(k+1)$ $\frac{11}{8} f(k)$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k)$ $f(k)$

$$\frac{3}{8} = \frac{1}{\frac{8}{3}} f(k)$$

$f(k+1)$ $\frac{8}{3}$ $\mathbf{28}$

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$f(4k)$ $f(2k)$ $f(k)$

$\mathbf{28}$ $\mathbf{14}$ $\mathbf{7}$

$$\frac{2}{3} = \frac{1}{\frac{3}{2}} f(k)$$

$f(k+1)$ $\frac{3}{2}$ $\mathbf{6}$

$\mathbf{7}$

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$f(2k)$ $f(k)$

$\mathbf{6}$ $\mathbf{3}$

$$\frac{1}{2} = \frac{1}{\frac{2}{1}} f(k)$$

$f(k+1)$ $\frac{2}{1}$ $\mathbf{2}$

$\mathbf{3}$

$$\frac{2}{1} = 1 + 1$$

$f(\mathbf{2})$ $f(\mathbf{1})$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$

$$\frac{8}{11} = \frac{1}{11} f(k)$$

$f(k+1)$ $\frac{1}{8} f(k)$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k)$ $f(k)$

$$\frac{3}{8} = \frac{1}{8} f(k)$$

$f(k+1)$ $\frac{1}{3} f(k)$

29 **28**

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$f(4k)$ $f(2k)$ $f(k)$

28 **14** **7**

$$\frac{2}{3} = \frac{1}{3} f(k)$$

$f(k+1)$ $\frac{1}{2} f(k)$

7 **6**

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$f(2k)$ $f(k)$

6 **3**

$$\frac{1}{2} = \frac{1}{2} f(k)$$

$f(k+1)$ $\frac{1}{1} f(k)$

3 **2**

$$\frac{2}{1} = 1 + 1$$

$f(2)$ $f(1)$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$

$$\frac{8}{11} = \frac{1}{11} f(k)$$

$f(k+1)$ $\frac{1}{8} f(k)$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k)$ $f(k)$

29

$$\frac{3}{8} = \frac{1}{8} f(k)$$

$f(k+1)$ $\frac{1}{3} f(k)$

29 **28**

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$f(4k)$ $f(2k)$ $f(k)$

28 **14** **7**

$$\frac{2}{3} = \frac{1}{3} f(k)$$

$f(k+1)$ $\frac{1}{2} f(k)$

7 **6**

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$f(2k)$ $f(k)$

6 **3**

$$\frac{1}{2} = \frac{1}{2} f(k)$$

$f(k+1)$ $\frac{1}{1} f(k)$

3 **2**

$$\frac{2}{1} = 1 + 1$$

$f(2)$ $f(1)$

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$

$$\frac{8}{11} = \frac{1}{11} f(k)$$

$f(k+1)$ $\frac{1}{8} f(k)$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k)$ $f(k)$

58 **29**

$$\frac{3}{8} = \frac{1}{8} f(k)$$

$f(k+1)$ $\frac{1}{3} f(k)$

29 **28**

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$f(4k)$ $f(2k)$ $f(k)$

28 **14** **7**

$$\frac{2}{3} = \frac{1}{3} f(k)$$

$f(k+1)$ $\frac{1}{2} f(k)$

7 **6**

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$f(2k)$ $f(k)$

6 **3**

$$\frac{1}{2} = \frac{1}{2} f(k)$$

$f(k+1)$ $\frac{1}{1} f(k)$

3 **2**

$$\frac{2}{1} = 1 + 1$$

$f(2)$ $f(1)$

2 **1**

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$

$$\frac{8}{11} = \frac{1}{11} f(k)$$

$f(k+1)$ $\frac{1}{8}$ **58**

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k)$ $f(k)$

58 **29**

$$\frac{3}{8} = \frac{1}{8} f(k)$$

$f(k+1)$ $\frac{1}{3}$ **28**

29

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$f(4k)$ $f(2k)$ $f(k)$

28 **14** **7**

$$\frac{2}{3} = \frac{1}{3} f(k)$$

$f(k+1)$ $\frac{1}{2}$ **6**

7

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$f(2k)$ $f(k)$

6 **3**

$$\frac{1}{2} = \frac{1}{2} f(k)$$

$f(k+1)$ $\frac{1}{1}$ **2**

3

$$\frac{2}{1} = 1 + 1$$

$f(2)$ $f(1)$

2

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$

$$\frac{8}{11} = \frac{1}{11} f(k)$$

$f(k+1)$ $\frac{1}{8}$ **58**

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k)$ $f(k)$

58 **29**

$$\frac{3}{8} = \frac{1}{8} f(k)$$

$f(k+1)$ $\frac{1}{3}$ **28**

29

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$f(4k)$ $f(2k)$ $f(k)$

28 **14** **7**

$$\frac{2}{3} = \frac{1}{3} f(k)$$

$f(k+1)$ $\frac{1}{2}$ **6**

7

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$f(2k)$ $f(k)$

6 **3**

$$\frac{1}{2} = \frac{1}{2} f(k)$$

$f(k+1)$ $\frac{1}{1}$ **2**

3

$$\frac{2}{1} = 1 + 1$$

$f(2)$ $f(1)$

2

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$
59

$$\frac{8}{11} = \frac{1}{11} f(k)$$

$f(k+1)$ $\frac{1}{8}$ $f(k)$
59 **58**

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k)$ $f(k)$
58 **29**

$$\frac{3}{8} = \frac{1}{8} f(k)$$

$f(k+1)$ $\frac{1}{3}$ $f(k)$
29 **28**

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$f(4k)$ $f(2k)$ $f(k)$
28 **14** **7**

$$\frac{2}{3} = \frac{1}{3} f(k)$$

$f(k+1)$ $\frac{1}{2}$ $f(k)$
7 **6**

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$f(2k)$ $f(k)$
6 **3**

$$\frac{1}{2} = \frac{1}{2} f(k)$$

$f(k+1)$ $\frac{1}{1}$ $f(k)$
3 **2**

$$\frac{2}{1} = 1 + 1$$

$f(2)$ $f(1)$
2 **1**

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$
118 **59**

$$\frac{8}{11} = \frac{1}{11} f(k)$$

$f(k+1)$ $\frac{1}{8}$ $f(k)$
59 **58**

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(2k)$ $f(k)$
58 **29**

$$\frac{3}{8} = \frac{1}{8} f(k)$$

$f(k+1)$ $\frac{1}{3}$ $f(k)$
29 **28**

$$\frac{8}{3} = 1 + \frac{5}{3} = 1 + 1 + \frac{2}{3}$$

$f(4k)$ $f(2k)$ $f(k)$
28 **14** **7**

$$\frac{2}{3} = \frac{1}{3} f(k)$$

$f(k+1)$ $\frac{1}{2}$ $f(k)$
7 **6**

$$\frac{3}{2} = 1 + \frac{1}{2}$$

$f(2k)$ $f(k)$
6 **3**

$$\frac{1}{2} = \frac{1}{2} f(k)$$

$f(k+1)$ $\frac{1}{1}$ $f(k)$
3 **2**

$$\frac{2}{1} = 1 + 1$$

$f(2)$ $f(1)$
2 **1**

從範例測資開始分析

$$\frac{30}{11} = 1 + \frac{19}{11} = 1 + 1 + \frac{8}{11}$$

$f(4k)$ $f(2k)$ $f(k)$
236 **118** **59**

$$\frac{8}{11} = \frac{1}{11} f(k)$$

$f(k+1)$ $\frac{1}{8} f(k)$
59 **58**

$$\frac{11}{8} = 1 + \frac{3}{8}$$

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$f(4k)$ $f(2k)$ $f(k)$
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$$\frac{2}{1} = 1 + 1$$

$f(2)$ $f(1)$
2 **1**

遞迴設計

遞迴設計

fact(n)

遞迴設計

$\text{fact}(n) = n * \text{fact}(n-1)$

遞迴設計

$\text{fact}(n) = n * \text{fact}(n-1)$

5!

遞迴設計

$\text{fact}(n) = n * \text{fact}(n-1)$

6 5!

遞迴設計

$\text{fact}(n) = n * \text{fact}(n-1)$

6! **6** **5!**

遞迴設計

$\text{fact}(n) = n * \text{fact}(n-1)$

6! **6** **5!**

$$\frac{11}{8}$$

遞迴設計

$\text{fact}(n) = n * \text{fact}(n-1)$

6! **6** **5!**

$$\frac{11}{8} = 1 + \frac{3}{8}$$

遞迴設計

$$\text{fact}(n) = n * \text{fact}(n-1)$$

$$6! \quad 6 \quad 5!$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(\mathbf{k})$

遞迴設計

$$\text{fact}(n) = n * \text{fact}(n-1)$$

$$6! \quad 6 \quad 5!$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$
$$f(\mathbf{2k}) \quad f(\mathbf{k})$$

遞迴設計

$$\text{fact}(n) = n * \text{fact}(n-1)$$

$$6! \quad 6 \quad 5!$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(\mathbf{2k}) \quad f(\mathbf{k})$
 $\mathbf{29}$

遞迴設計

$$\text{fact}(n) = n * \text{fact}(n-1)$$

$$6! \quad 6 \quad 5!$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$
$$f(\mathbf{2k}) \quad f(\mathbf{k})$$
$$\mathbf{58} \quad \mathbf{29}$$

遞迴設計

$$\text{fact}(n) = n * \text{fact}(n-1)$$

$$6! \quad 6 \quad 5!$$

 findK(a,b)
a>b

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$f(\mathbf{2k})$ $f(\mathbf{k})$
58 **29**

遞迴設計

$$\text{fact}(n) = n * \text{fact}(n-1)$$

$$6! \quad 6 \quad 5!$$

$$\left\{ \begin{array}{l} \text{findK}(a,b) = 2 * \text{findK}(a-b,b) \\ \quad \quad \quad a > b \end{array} \right.$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$
$$\begin{array}{cc} f(\mathbf{2k}) & f(\mathbf{k}) \\ \mathbf{58} & \mathbf{29} \end{array}$$

遞迴設計

$$\text{fact}(n) = n * \text{fact}(n-1)$$

$$6! \quad 6 \quad 5!$$

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遞迴設計

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$$\frac{11}{8} = 1 + \frac{3}{8}$$
$$\begin{array}{cc} f(\mathbf{2k}) & f(\mathbf{k}) \\ \mathbf{58} & \mathbf{29} \end{array}$$
$$\frac{3}{8} = \frac{1}{\frac{8}{3}}$$

遞迴設計

$$\text{fact}(n) = n * \text{fact}(n-1)$$

$$6! \quad 6 \quad 5!$$

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$$\frac{11}{8} = 1 + \frac{3}{8}$$
$$\begin{array}{cc} f(\mathbf{2k}) & f(\mathbf{k}) \\ \mathbf{58} & \mathbf{29} \end{array}$$
$$\frac{3}{8} = \frac{1}{8} \frac{1}{3} f(\mathbf{k})$$

遞迴設計

$$\text{fact}(n) = n * \text{fact}(n-1)$$

$$6! \quad 6 \quad 5!$$

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$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$\begin{array}{cc} f(\mathbf{2k}) & f(\mathbf{k}) \\ \mathbf{58} & \mathbf{29} \end{array}$$

$$\frac{3}{8} = \frac{1}{8}$$

$$\begin{array}{cc} f(\mathbf{k+1}) & \frac{1}{3} f(\mathbf{k}) \end{array}$$

遞迴設計

$$\text{fact}(n) = n * \text{fact}(n-1)$$

$$6! \quad 6 \quad 5!$$

$$\left\{ \begin{array}{l} \text{findK}(a,b) = 2 * \text{findK}(a-b,b) \\ \quad \quad \quad a > b \end{array} \right.$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$\begin{array}{cc} f(2k) & f(k) \\ 58 & 29 \end{array}$$

$$\frac{3}{8} = \frac{1}{8}$$

$$\begin{array}{cc} f(k+1) & f(k) \\ 3 & 28 \end{array}$$

遞迴設計

$$\text{fact}(n) = n * \text{fact}(n-1)$$

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遞迴設計

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$$\left\{ \begin{array}{l} \text{findK}(a,b) = 2 * \text{findK}(a-b,b) \\ \quad \quad \quad a > b \\ \\ \text{findK}(a,b) \\ \quad \quad \quad a < b \end{array} \right.$$

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遞迴設計

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$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$\begin{array}{cc} f(\mathbf{2k}) & f(\mathbf{k}) \\ \mathbf{58} & \mathbf{29} \end{array}$$

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遞迴設計

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$$\begin{array}{cc} f(\mathbf{2k}) & f(\mathbf{k}) \\ \mathbf{58} & \mathbf{29} \end{array}$$

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$$\begin{array}{cc} f(\mathbf{k+1}) & \frac{1}{3} f(\mathbf{k}) \\ \mathbf{29} & \mathbf{28} \end{array}$$

$$f(\mathbf{1})$$

遞迴設計

$$\text{fact}(n) = n * \text{fact}(n-1)$$

$$6! \quad 6 \quad 5!$$

$$\left\{ \begin{array}{ll} \text{findK}(a,b) & = 2 * \text{findK}(a-b,b) \\ & \text{a>b} \\ \text{findK}(a,b) & = 1 + \text{findK}(b,a) \\ & \text{a<b} \end{array} \right.$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$\begin{array}{cc} f(\mathbf{2k}) & f(\mathbf{k}) \\ \mathbf{58} & \mathbf{29} \end{array}$$

$$\frac{3}{8} = \frac{1}{8}$$

$$\begin{array}{cc} f(\mathbf{k+1}) & \frac{1}{3} f(\mathbf{k}) \\ \mathbf{29} & \mathbf{28} \end{array}$$

$$f(\mathbf{1}) = \mathbf{1}$$

遞迴設計

$$\text{fact}(n) = n * \text{fact}(n-1)$$

$$6! \quad 6 \quad 5!$$

$$\left\{ \begin{array}{ll} \text{findK}(a,b) & = 2 * \text{findK}(a-b,b) \\ & \mathbf{a > b} \\ \text{findK}(a,b) & = 1 + \text{findK}(b,a) \\ & \mathbf{a < b} \\ \text{findK}(a,b) & \\ & \mathbf{a == b} \end{array} \right.$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$
$$\begin{array}{cc} f(\mathbf{2k}) & f(\mathbf{k}) \\ \mathbf{58} & \mathbf{29} \end{array}$$

$$\frac{3}{8} = \frac{1}{8}$$
$$\begin{array}{cc} f(\mathbf{k+1}) & \frac{1}{3} f(\mathbf{k}) \\ \mathbf{29} & \mathbf{28} \end{array}$$

$$f(\mathbf{1}) = \mathbf{1}$$

遞迴設計

$$\text{fact}(n) = n * \text{fact}(n-1)$$

$$6! \quad 6 \quad 5!$$

$$\left\{ \begin{array}{ll} \text{findK}(a,b) &= 2 * \text{findK}(a-b,b) \\ &\text{a>b} \\ \text{findK}(a,b) &= 1 + \text{findK}(b,a) \\ &\text{a<b} \\ \text{findK}(a,b) &= 1 \\ &\text{a==b} \end{array} \right.$$

$$\frac{11}{8} = 1 + \frac{3}{8}$$

$$\begin{array}{cc} f(\mathbf{2k}) & f(\mathbf{k}) \\ \mathbf{58} & \mathbf{29} \end{array}$$

$$\frac{3}{8} = \frac{1}{8}$$

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