Programming Examples Using Arrays

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Search and Sort an Array

- Two common problems in processing arrays
  - **Searching** an array to determine the location of a particular value.
  - **Sorting** an array to rearrange the array elements in numerical order.
- Examples
  - Search an array of student exam scores to determine which student, if any, got a particular score.
  - Rearrange the array elements in increasing (decreasing) order by score.
- Algorithm for searching over a sorted array is much more efficient than over an unsorted array.

Algorithm of Linear Search

(Sequential Search)

1. Assume the **target** has not been found.
2. Start with the initial array element.
3. Repeat while the target is not found and there are more array elements
   - 3.1 if the current element matches the target
     - 3.1.1 Set a flag to indicate that the target has been found

       else
     - 3.1.2 Advance to the next array element
4. if the target was found
   - 4.1 Return the target index as the search result

   else
   - 4.2 Return -1 as the search result

Search an Array

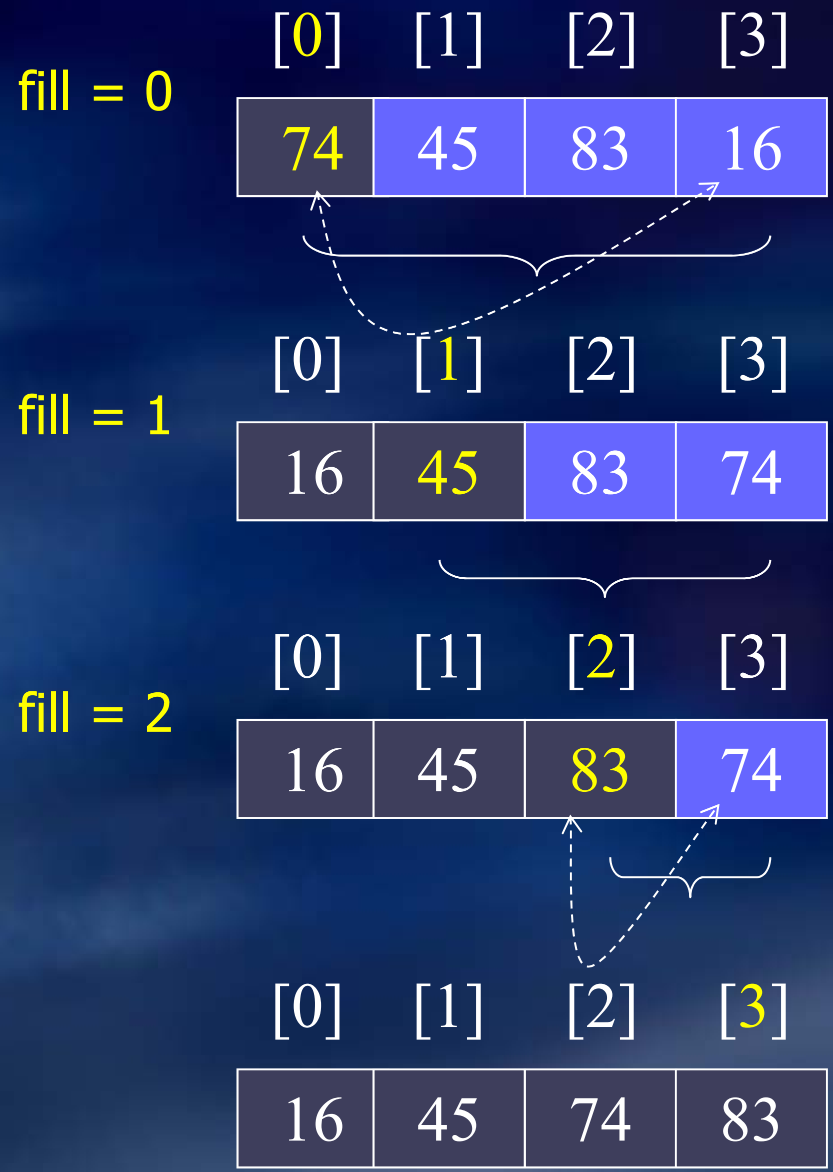
```
int search(const int array[],  /* input - array to search */
           int       target,   /* input - value searched for */
           int       n) {      /* input - number of elements to search */
    int i,
        found = 0,     /* whether or not target has been found */
        where;         /* index where target found or NOT_FOUND */

    /* Compares each element to target */
    i = 0;
    while (!found && i < n) {
       if (array[i] == target)
          found = 1;
       else
          ++i;
    }

    /* Returns index of element matching target or NOT_FOUND */
    if (found)
       where = i;
    else
       where = NOT_FOUND;

    return (where);
}
```

Sorting an Array



Selection sort is an intuitive sorting algorithm.

- Find the index of the smallest element in the array.
- Swap the smallest element with the first element.
- Repeat the above steps for the 2nd, 3rd, …, smallest elements.

Function select_sort

```
int get_min_range(int list[], int first, int last);
void select_sort(int list[],   /* input/output - array being sorted      */
                 int n)        /* input - number of elements to sort     */
{
      int fill,                /* first element in unsorted subarray */
          temp,                /* temporary storage                  */
          index_of_min;        /* subscript of next smallest element */

      for (fill = 0; fill < n-1; ++fill) {
         /* Find position of smallest element in the unsorted subarray */
          index_of_min = get_min_range(list, fill, n-1);

          /* Exchange elements at fill and index_of_min */
          if (fill != index_of_min) {
                temp = list[index_of_min];
                list[index_of_min] = list[fill];
                list[fill] = temp;
          }
      }
}
```

Computing Statistics

- Most common use of arrays is for storage of a collection of related data values.
- Once the values are stored, we can perform some simple statistical computations.

sum = x[0] + x[1] + … + x[MAX_ITEM-1]

mean = sum / MAX_ITEM

sum_square = x[0]^2 + x[1]^2 + … + x[MAX_ITEM-1]^2

variance = (sum_square – MAX_ITEM * mean^2) / (MAX_ITEM – 1)

standard deviation = sqrt(variance)

histogram?

mode?

median?

Computing Statistics (cont'd)

Figure 8.3

```
#include <stdio.h>
#include <math.h>
#define MAX_ITEM  8  /* maximum number of items in list of data     */
int
main(void)
{
   double x[MAX_ITEM],   /* data list                              */
          mean,          /* mean (average) of the data             */
          st_dev,        /* standard deviation of the data */
          sum,           /* sum of the data                        */
          sum_sqr;       /* sum of the squares of the data */
   int    i;

   /* Gets the data   */
   printf("Enter %d numbers separated by blanks or <return>s\n> ",
          MAX_ITEM);
   for  (i = 0;  i < MAX_ITEM;  ++i)
      scanf("%lf", &x[i]);
```

```
/* Computes the sum and the sum of the squares of all data  */
sum = 0;
sum_sqr = 0;
for (i = 0; i < MAX_ITEM; ++i) {
    sum += x[i];
    sum_sqr += x[i] * x[i];
}

/* Computes and prints the mean and standard deviation      */
mean = sum / MAX_ITEM;
st_dev = sqrt((sum_sqr - MAX_ITEM * mean * mean)
                                        / (MAX_ITEM-1));
printf("The mean is %.2f.\n", mean);
printf("The standard deviation is %.2f.\n", st_dev);

/* Displays the difference between each item and the mean   */
printf("\nTable of differences between data values and mean\n");
printf("Index       Item       Difference\n");
for (i = 0; i < MAX_ITEM; ++i)
    printf("%3d%4c%9.2f%5c%9.2f\n", i, ' ', x[i], ' ', x[i] - mean);

return (0);
}
```

Computing Statistics (cont'd)

Enter 8 numbers separated by blanks or <return>s
> **16 12 6 8 2.5 12 14 -54.5**

The mean is **2.00**.
The standard deviation is **21.75**.

Table of differences between data values and mean

Index	Item	Difference
0	16.00	14.00
1	12.00	10.00
2	6.00	4.00
3	8.00	6.00
4	2.50	0.50
5	12.00	10.00
6	14.00	12.00
7	-54.50	-56.50

Matrix Operations

- Addition
  - Ex. A and B are both 3-by-5, C = A + B

$$\begin{pmatrix} 1 & 2 & 3 & 2 & 3 \\ 4 & 5 & 6 & 5 & 6 \\ 1 & 2 & 5 & 4 & 5 \end{pmatrix} + \begin{pmatrix} 7 & 2 & 3 & 2 & 6 \\ 4 & 1 & 0 & 3 & 2 \\ 1 & 4 & 4 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 8 & 4 & 6 & 4 & 9 \\ 8 & 6 & 6 & 8 & 8 \\ 2 & 6 & 9 & 6 & 7 \end{pmatrix}$$

  - $C_{ij} = A_{ij} + B_{ij}$

```
int i, j, m=3, n=5;
double a_mat[3][5], b_mat[3][5];
double c_mat[3][5];
for (i=0; i<m; i++)
    for (j=0; j<n; j++)
        c_mat[i][j] = a_mat[i][j] + b_mat[i][j];
```

Matrix Operations (cont'd)

- Multiplication
  - Ex. A and B are both 3-by-5, $C = A\,B^T$

$$\begin{pmatrix} 1 & 2 & 3 & 2 & 3 \\ 4 & 5 & 6 & 5 & 6 \\ 1 & 2 & 5 & 4 & 5 \end{pmatrix} \begin{pmatrix} 7 & 4 & 1 \\ 2 & 1 & 4 \\ 3 & 0 & 4 \\ 2 & 3 & 2 \\ 6 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 42 & 18 & 31 \\ 102 & 48 & 70 \\ 64 & 28 & 47 \end{pmatrix}$$

  - $C_{ij} = \sum_{k=1}^{5} A_{ik}\, B^T_{kj}$

```
int i, j, k, m=3, n=5;
double a_mat[3][5], b_mat[3][5];
double c_mat[3][3];
for (i=0; i<m; i++)
    for (j=0; j<m; j++)
        for (k=0, c_mat[i][j]=0; k<n; k++)
            c_mat[i][j] += a_mat[i][k] * b_mat[j][k];
```

Matrix Operations (cont'd)

- In-place Computation??

  $A, B \in R^{mxn}$, $A + B \rightarrow A$; $A, B \in R^{nxn}$, $A\,B^{T} \rightarrow A$

```
int i, j, m=3, n=5;
double a_mat[3][5], b_mat[3][5];
for (i=0; i<m; i++)
    for (j=0; j<n; j++)
        a_mat[i][j] += b_mat[i][j];
```

```
int i, j, k, n=3;
double a_mat[3][3];
double b_mat[3][3], sum;
for (i=0; i<n; i++)
    for (j=0; j<n; j++)
    {
        for (k=0, sum=0; k<n; k++)
            sum += a_mat[i][k] * b_mat[j][k];
        a_mat[i][j] = sum;
    }
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 5 \end{pmatrix} \begin{pmatrix} 7 & 4 & 1 \\ 2 & 1 & 4 \\ 3 & 0 & 4 \end{pmatrix}$$

Gaussian Elimination

- Solve linear equations using Gaussian Elimination and find out the rank. e.g.

  L1: $2x + y - z = 8$
  L2: $-3x - y + 2z = -11$
  L3: $-2x + y + 2z = -3$

- In matrix notation, we have the following

$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & 8 \\ -3 & -1 & 2 & -11 \\ -2 & 1 & 2 & -3 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0.5 & -0.5 & 4 \\ 0 & 0.5 & 0.5 & 1 \\ 0 & 2 & 1 & 5 \end{array}\right)$$

Normalize 1st row
Multiply 3 and add to 2nd row
Multiply 2 and add to 3rd row

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -1 & 1 \end{array}\right)$$

Normalize 2nd row
Multiply -0.5 and add to 1st row
Multiply -2 and add to 3rd row

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

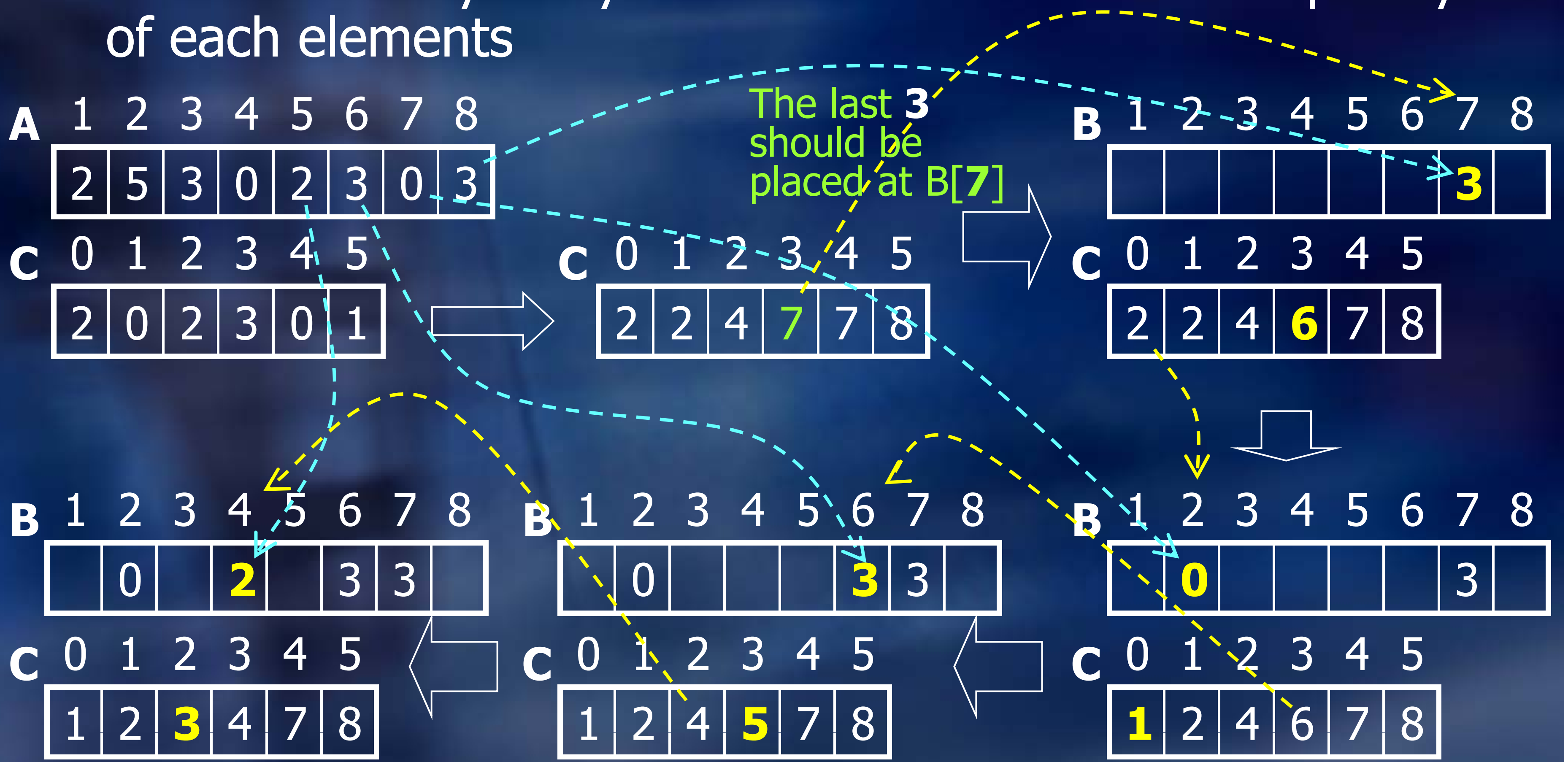
Normalize 3rd row
Multiply 1 and add to 1st row
Multiply -1 and add to 2nd row

x=2, y=3, z = -1 are the result.
Since the left 3 by 3 submatrix is an identity matrix, the number of independent equations is 3. (If one row is all zero, then the number of independent equations is 2 and if two rows are all zero, the number of independent equations is 1, etc.)

Counting Sort

LC1122 Relative Sort Array

- Elements to be sorted are in a set {0,1,…,k}
- Use an auxiliary array to count the occurrence frequency of each elements



- Non-comparison sort, stable sort, O(n)

Radix Sort

- Stably sort each digits, least significant digits first

326	690	704	326
453	751	608	435
608	453	326	453
835	704	835	608
751	835	435	690
435	435	751	704
704	326	453	751
690	608	690	835

sorted

Bucket (Bin) Sort
distribute MSB first

- Radix-sort(Array, n)
  1. for i=0 to n-1
  2. use a stable sort algorithm to sort Array on digit i

Radix-8 Sort (cont’d)

- A radix-8 sort
  - 1-dim array of positive integers to be sorted:
    - e.g. 10**0**, 00**3**, 66**7**, 02**7**, 12**0**, 01**3**, 32**5** **in octal**
  - 2-dim array of integers is used as the working space
    - rows (the buckets) indexed from 0 to 7 and
    - columns indexed from 0 to n-1

	0	1	2			n-1
0	100	120				
1						
2						
3	003	013			…	
4						
5	325					
6						
7	667	027				

100	120	003	013	325	667	027

	0	1	2			n-1
0	100	003				
1	013					
2	120	325	027			
3					…	
4						
5						
6	667					
7						

100	003	013	120	325	027	667

	0	1	2			n-1
0	003	013	027			
1	100	120				
2	325					
3					…	
4						
5						
6	667					
7						

003	013	027	100	120	325	667

Radix-8 Sort (cont'd)

- The radix-8 sorting is done as follows:
  - **Distribute:** Place each value of the one-dimensional vector into a bucket, based on the value's rightmost octal digit. For example, 67 is placed in row 7, 3 is placed in row 3 and 100 is placed in row 0. This procedure is called a distribution pass.
  - **Gather**: Loop through the bucket vector row by row, and copy the values back to the original vector. This procedure is called a gathering pass. The new order of the preceding values in the one-dimensional vector is 100, 3 and 67.
  - **Repeat** this process for each subsequent digit position (2nd rightmost, 3rd rightmost, etc.). e.g. On the second pass, 100 is placed in row 0, 3 is placed in row 0 (3 can be seen as 003) and 97 is placed in row 9. After the gathering pass, the order of the values in the one-dimensional vector is 100, 3 and 97. On the third (3rd rightmost) pass, 100 is placed in row 1, 3 is placed in row 0 and 97 is placed in row 0 (after the 3). After this last gathering pass, the original vector is in sorted order.

Radix Sort Implementation

```c
void radix8Sort(int ndata, int data[]) {
    int buckets[8][MAX], int nBucket[8];
    int i, j, k, index, mult, iBucket;
    int len = maxNumDigits(ndata, data); /* max number of octal digits */
    mult = 1;
    for (i=0; i<len; i++) {
        for (j=0; j<8; j++) nBucket[j] = 0;
        for (j=0; j<ndata; j++) {
            iBucket = data[j] / mult % 8;
            buckets[iBucket][nBucket[iBucket]++] = data[j];
        }
        for (j=0, index=0; j<8; j++)
            for (k=0; k<nBucket[j]; k++)
                data[index++] =
                            buckets[j][k];
        mult *= 8;
    }
}
```

redistribute

gather

```c
int maxNumDigits(int ndata,
                         int data[]) {
    int i, max = -1;
    for (i=0; i<ndata; i++)
        if (data[i] > max)
            max = data[i];
    return (log10(max)/log10(8))+1;
}
```

Parallel Arrays

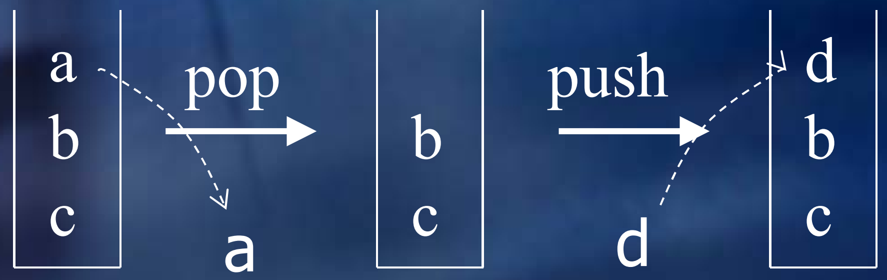
- Two or more arrays with the same number of elements used for **storing related information** about a collection of data objects
- A very common method to organize data with arrays

id[0]	5503	gpa[0]	2.71
id[1]	4556	gpa[1]	3.09
id[2]	5691	gpa[2]	2.98
	…		…
id[49]	9146	gpa[49]	1.92

- id[**i**] and gpa[**i**] refer to the information related to the **i**-th student

Stacks

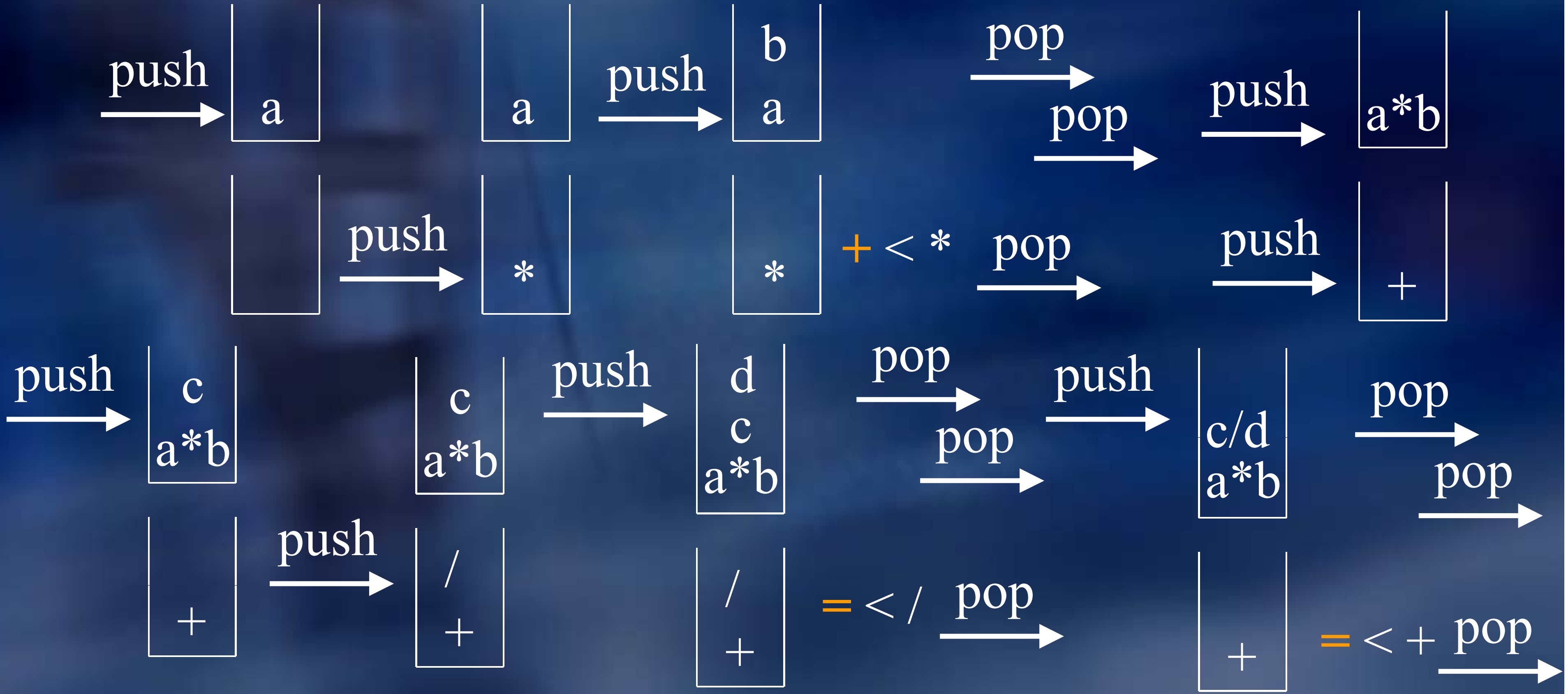
- A **stack** is a data structure in which only the top element can be accessed.
- For example, the plates stored in the spring-loaded device in a buffet line perform like a stack. A customer always takes the top plate; when a plate is removed, the plate beneath it moves to the top.
- Popping the stack: remove a value from a stack.
- Pushing it onto the stack: store an item in a stack.



- Array is one of the approaches to implement a stack.

Algorithm Utilizing Stacks

- Expression evaluation
  a * b + c / d =
  - Two stacks: operand stack, operator stack



Push: Insert a New Element to the Top of Stack

```
#define STACK_SIZE 100
char stack[STACK_SIZE];
int top = -1; /* the position of current stack top */
```

```
push(stack, 'a', &top, STACK_SIZE);
```

```
void
push(char stack[],      /* input/output - the stack */
     char item,         /* input - data being pushed onto the stack */
     int  *top,         /* input/output - pointer to top of stack */
     int  max_size)     /* input - maximum size of stack */
{
    if (*top < max_size-1) {
        ++(*top);
        stack[*top] = item;
    }
}
```

Pop: Remove from Top of Stack an Element

```
char content;
...
content = pop(stack, &top);
```

```
char
pop(char stack[],   /* input/output - the stack */
        int  *top)       /* input/output - pointer to top of stack */
{
      char item;      /* value popped off the stack */

      if (*top >= 0) {
            item = stack[*top];
            --(*top);
      } else {
            item = STACK_EMPTY;
      }
      return item;
}
```

Binary Tree

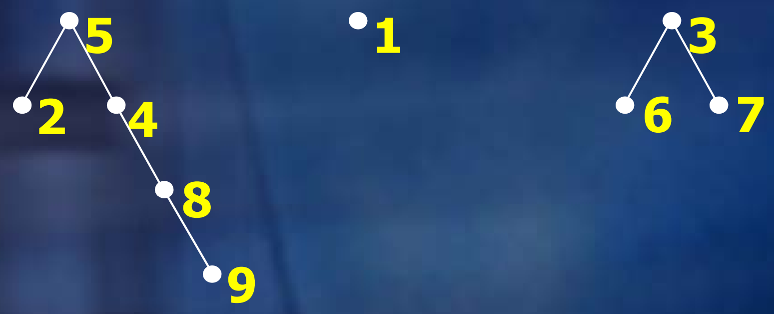
- A binary tree is a tree data structure in which each node has at most two children.

104	71	24	66	27	23	8	5	32	NIL	25	18	22	NIL	NIL
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

- To represent a binary tree, the value in node label i can be stored in cell i of an array
- The parent of node label i is node label $\lfloor i/2 \rfloor$
  The left child of node label i is node label 2*i
  The right child of node label i is node label 2*i+1

Disjoint Set

- Pairwise disjoint sets X and Y satisfies $X \cap Y = \phi$
  e.g. {2, 4, 5, 8, 9}, {1}, {3, 6, 7}
- Each set can be represented as an arbitrary structured tree and the root is marked as the representative of that set



- This disjoint sets can be represented as an array – parent, parent[i] is the parent of i. A root's parent is itself.

parent	**1**	**5**	**3**	**5**	**5**	**3**	**3**	**4**	**8**
	1	2	3	4	5	6	7	8	9