Sudoku, Mathdoku, and Related Problems

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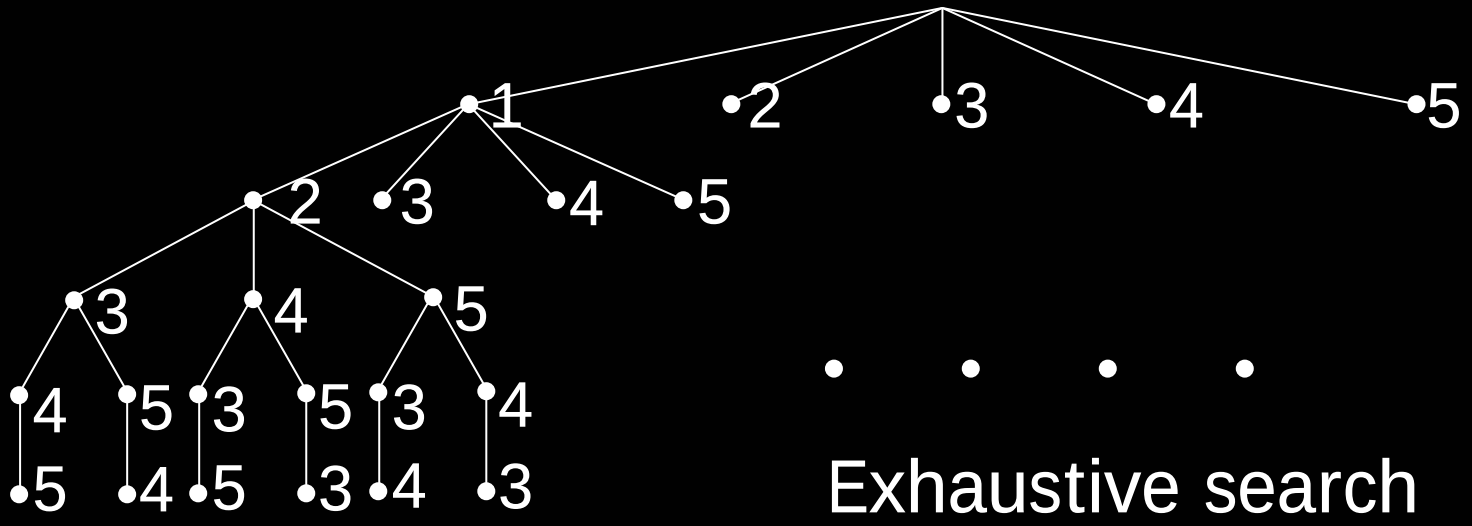
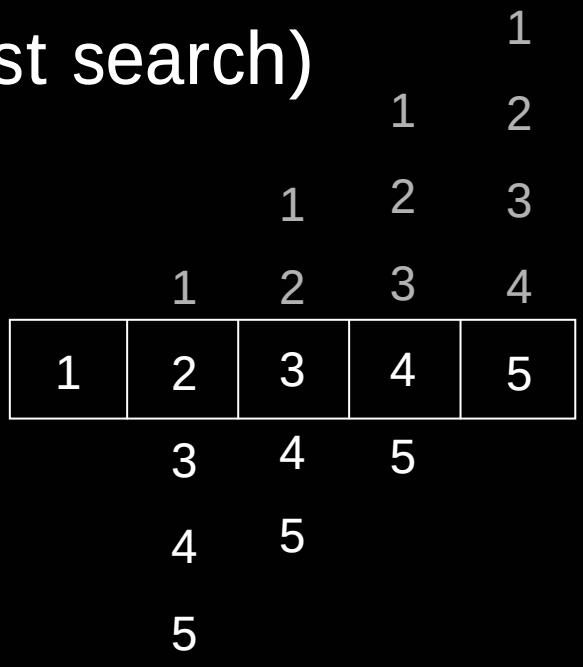
Generating Permutations



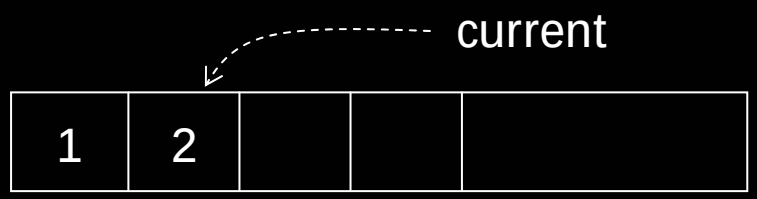
Exhaustive search
(depth first search)

5! = 120
permutations

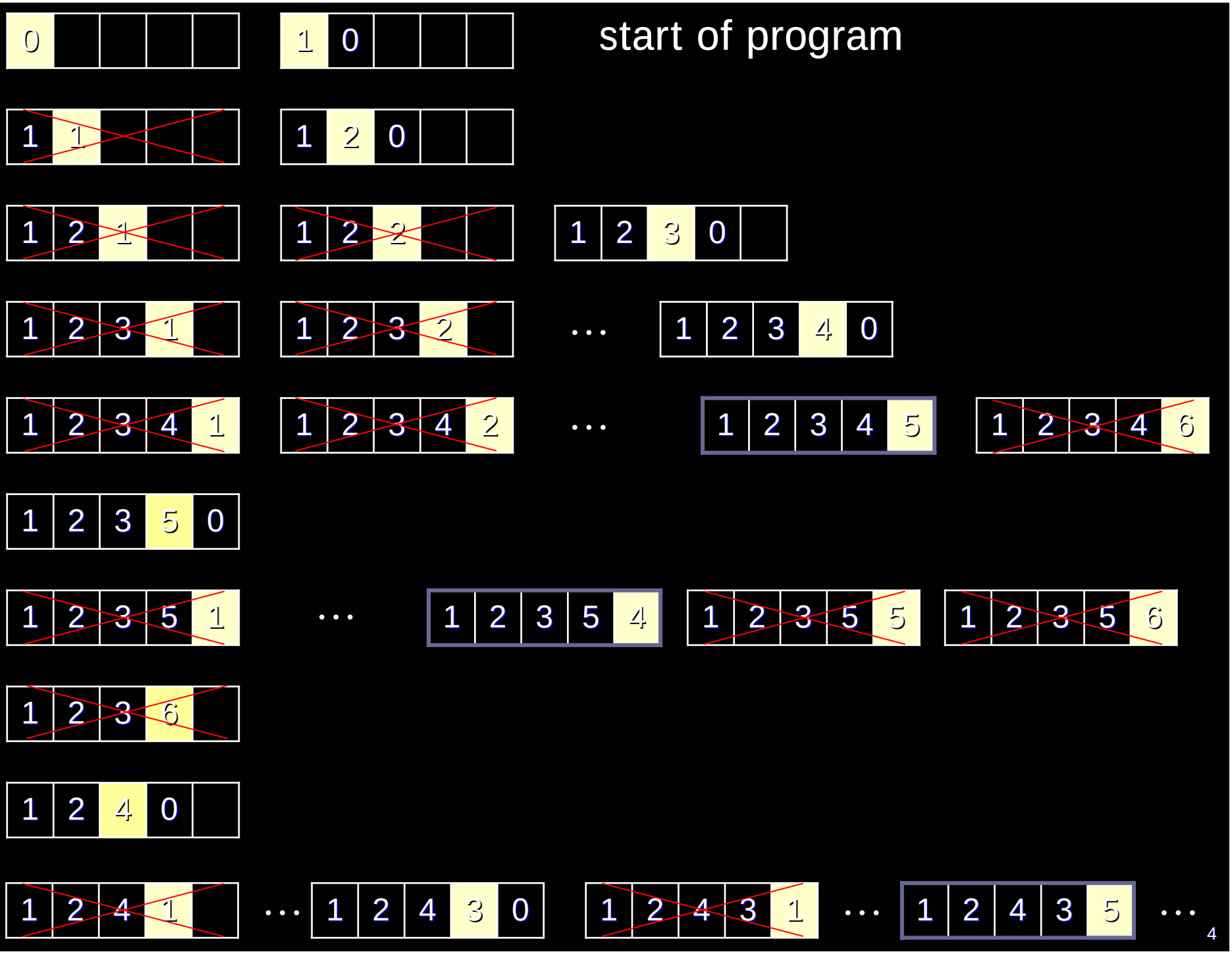
1 2 3 4 5
1 2 3 5 4
1 2 4 3 5
1 2 4 5 3
1 2 5 3 4
1 2 5 4 3
…

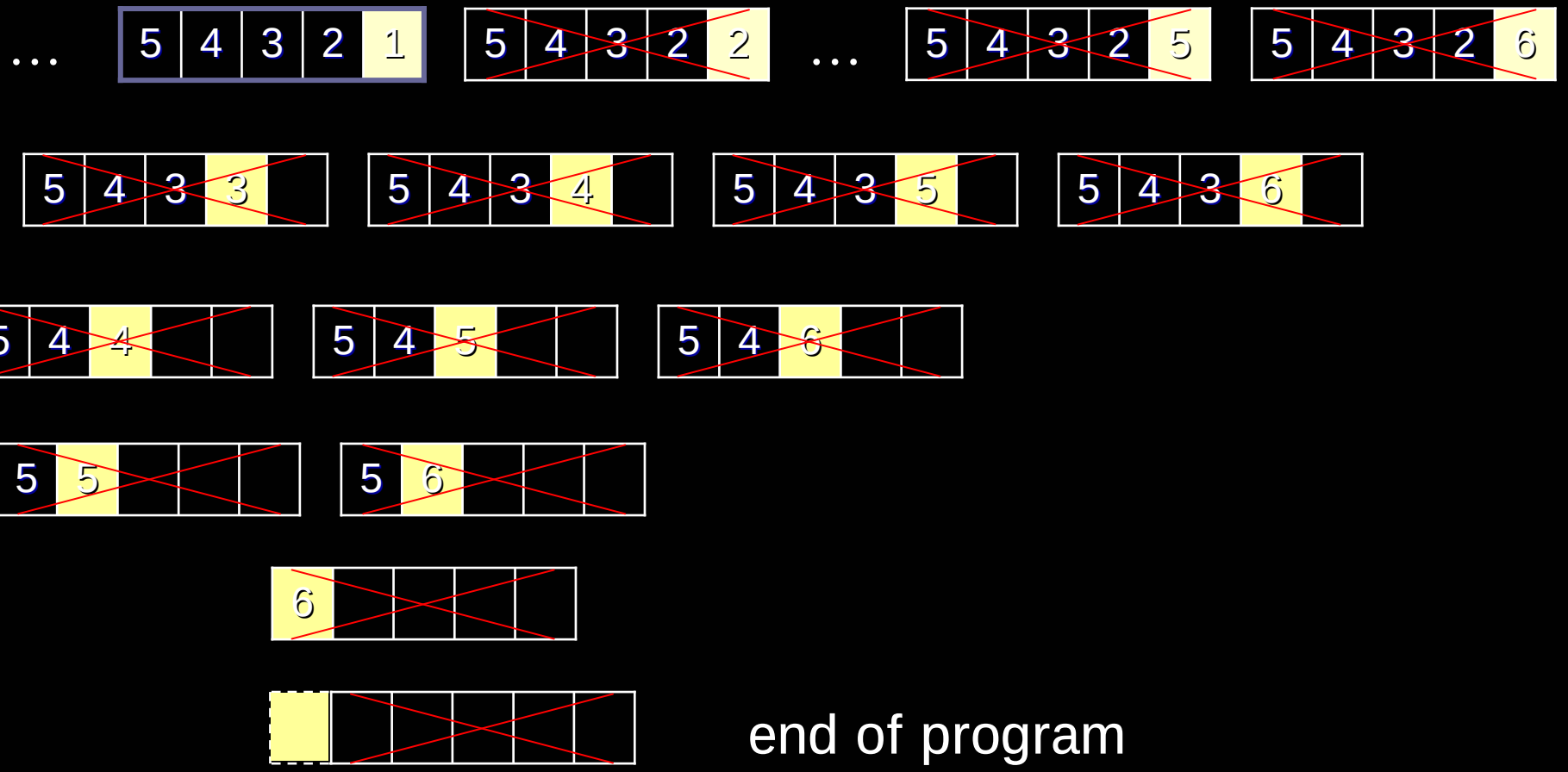


```
01 #include <stdio.h>
02
03 void main()
04 {
05     int size, perm[12] = {0}, current=0, solCount=0, i;
06
07     printf("Please input number of elements: ");
08     scanf("%d", &size);
09
10     while (current>=0)
11     {
12         current += next(size, current, perm);
13         if (current == size)
14         {
15             solCount++;
16             printf("%4d: ", solCount);
17             for (i=0; i<size; i++)
18                 printf("%d ", perm[i]);
19             printf("\n");
20             current = size-1;
21         }
22     }
23     printf("Total %d permutations\n",solCount);
24 }
```




```
26 int next(int size, int pivot, int perm[]
27 {
28     int i, collision;
29
30     while (perm[pivot]++ < size)
31     {
32         collision = 0;
33         for (i=0; i<pivot; i++)
34             if (perm[pivot] == perm[i])
35             {
36                 collision = 1;
37                 break;
38             }
39         if (!collision) return 1;
40     }
41     perm[pivot] = 0;
42     return -1;
43 }
```


start of program
0
1 0
1 1
1 2 0
1 2 1
1 2 2
1 2 3 0
1 2 3 1
1 2 3 2
1 2 3 4 0
1 2 3 4 1
1 2 3 4 2
1 2 3 4 5
1 2 3 4 6
1 2 3 5 0
1 2 3 5 1
1 2 3 5 4
1 2 3 5 5
1 2 3 5 6
1 2 3 6
1 2 4 0
1 2 4 1
1 2 4 3 0
1 2 4 3 1
1 2 4 3 5


...
5 4 3 2 1
5 4 3 2 2
...
5 4 3 2 5
5 4 3 2 6
5 4 3 3
5 4 3 4
5 4 3 5
5 4 3 6
5 4 4
5 4 5
5 4 6
5 5
5 6
6
end of program

Sudoku

- **Sudoku:** In these three examples, 81 cells are divided into 9 blocks each with 9 cells (3-by-3). A player is required to fill in the blank cells such that integers in each row, each column, and each block are permutations of {1,2,3,…,9}, i.e. no duplication of numbers in each row, column, or block.

		6	1		3	4		
		3			8			
5	4		7			1	2	
			2					4
	5		3		9		7	
7					4			
	3	7			5		4	2
			8			7		
		1	4		7	8		

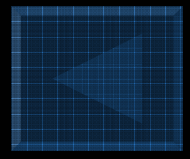
3		8				6		
	6					4		
	9		8	5			1	
2			9					
4		5				9		2
					7			6
	2			1	3		6	
		9					3	
		1				5		4

7	8	9		2				5
6					5		8	
							1	
2			8				5	1
		5	7		1	6		
9	1				6			3
	7							
	5		9					6
8				1		5	3	7

number of lines
row, column, value
row, column, value
…

```
30
0, 0, 7
0, 1, 8
0, 2, 9
0, 4, 2
0, 8, 5
1, 0, 6
1, 5, 5
1, 7, 8
…
```

Initial configuration



Data Representation

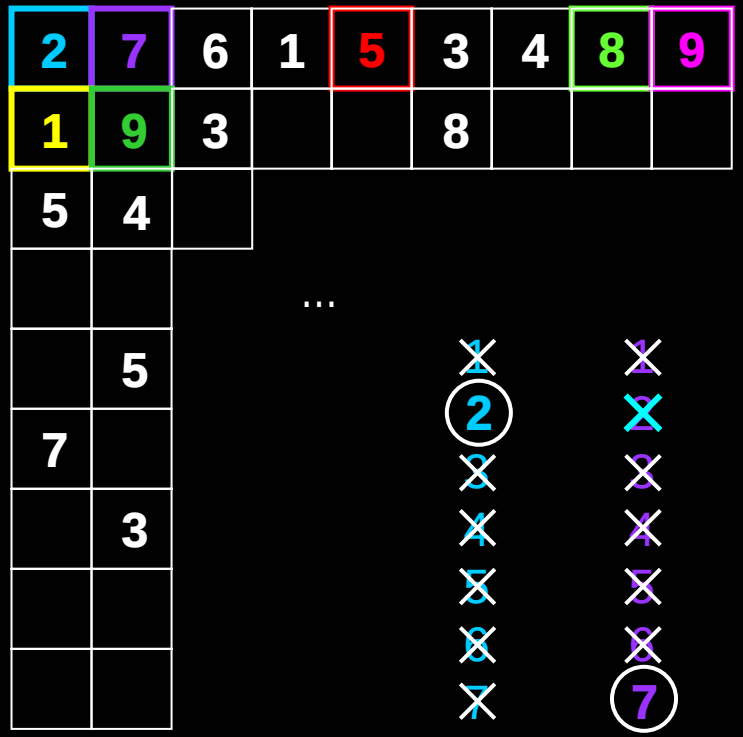
- Two dimensional integer array: int board[9][9]; initialized with 0's and fixed constraints

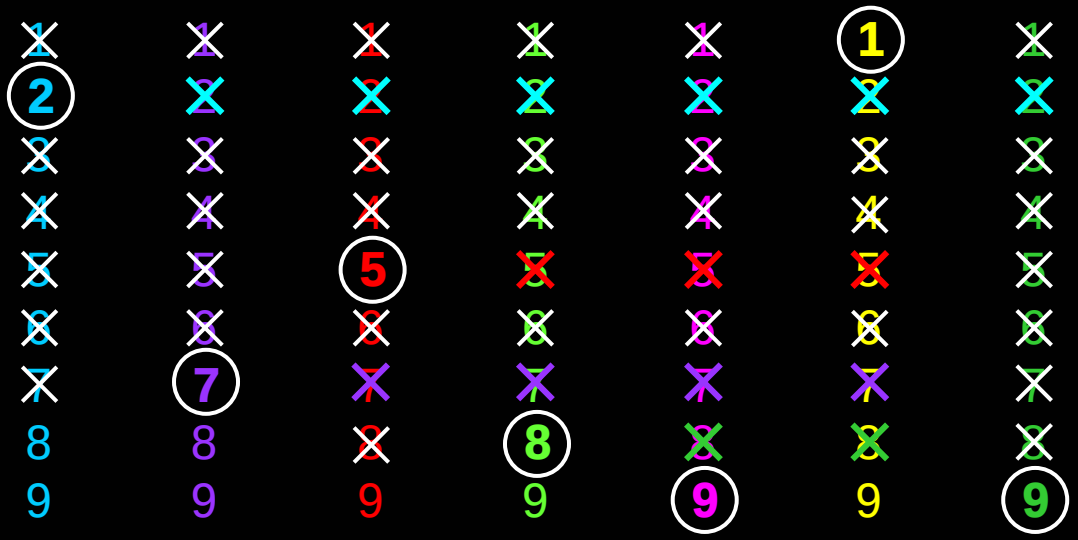
0	0	**6**	**1**	0	**3**	**4**	0	0
0	0	**3**	0	0	**8**	0	0	0
5	**4**	0	**7**	0	0	**1**	**2**	0
0	0	0	**2**	0	0	0	0	**4**
0	**5**	0	**3**	0	**9**	0	**7**	0
7	0	0	0	0	**4**	0	0	0
0	**3**	**7**	0	0	**5**	0	**4**	**2**
0	0	0	**8**	0	0	**7**	0	0
0	0	**1**	**4**	0	**7**	**8**	0	0

Depth First Search Process

- Extension of permutation generation: more constraints on the set of numbers to be filled in each cell

		6	1		3	4		
		3			8			
5	4		7			1	2	
			2					4
	5		3		9		7	
7					4			
	3	7			5		4	2
			8			7		
		1	4		7	8		





Satisfying Constraints

- For each cell
  - No two cells are assigned the same value in each row

```
for (i=0; i<9; i++)
    if (value == board[i][icol]) return 0;
```

  - No two cells are assigned the same value in each column

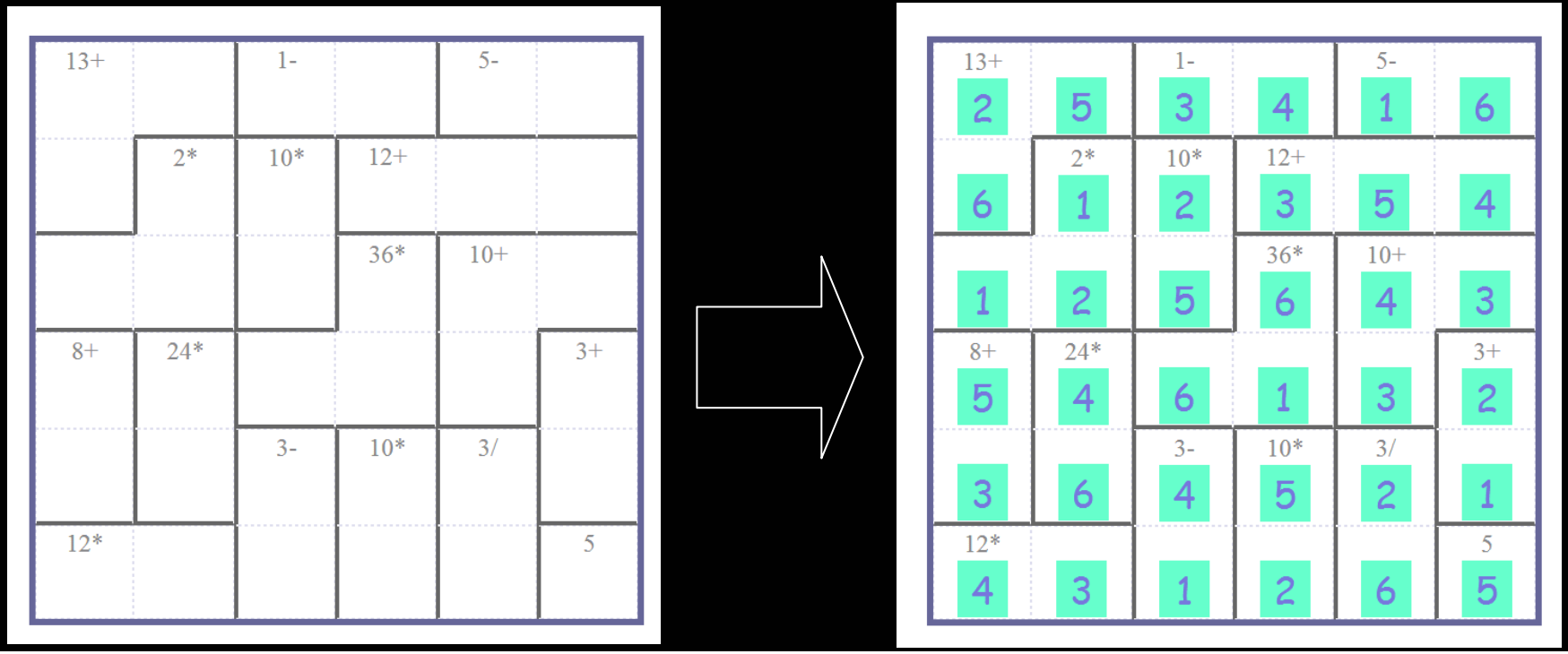
```
for (i=0; i<9; i++)
    if (value == board[irow][i]) return 0;
```

  - No two cells are assigned the same value in each 3x3 subblock

```
for (i=irow/3*3; i<irow/3*3+3; i++)
    for (j=icol/3*3; j<icol/3*3+3; j++)
        if (value == board[i][j]) return 0;
```

Mathdoku

- 4x4, 6x6, 8x8, …, nxn
- The values in each cell are from the set {1, 2, …, n}
- No repetition is allowed in each row and each column
- In addition, the values in each non-regular subblocks should satisfy the labeled arithmetic constraints, e.g.
  13+ is satisfied by 2+5+6=13, 1- is satisfied by 4-3=1,
  36* is satisfied by 6*6*1=36, and 3/ is satisfied by 6/2=3



Data Representation

1. Two dimensional integer array to store the chosen numbers.
2. Two dimensional integer array to store the constraint subblocks.

For example,

```
6
+  13     3  1 2 7
-   1     2  3 4
-   5     2  5 6
*   2     3  8 13 14
*   10    2  9 15
...
$
```

0	1	0	2	0	5
2	0	0	0	10	11
8	13	9	0	0	17
0	0	16	21	18	0
19	20	0	0	0	24
0	31	27	28	29	0

+		-		-	
	*	*	+		
			*	+	
+	*				+
		-	*	/	
*					5

13		1		5	
	2	10	12		
			36	10	
8	24				3
		3	10	3	
12					5

0	0	0	**2**01	0	**2**05
113	0	0	0	0	**1**12
0	**3**02	**3**10	0	0	0
0	0	0	**3**36	**1**10	0
108	**3**24	0	0	0	**1**03
0	**3**12	**2**03	**3**10	**4**03	5

Constraints are checked at the end of a subblock

Basic Algorithm

- Enumerating all cells with {1, 2, …, n} starting from the cell in the left-top corner, from top to down, from left to right, i.e.
- Satisfying row and column uniqueness constraints
- Skipping all cells with fixed numbers
- For the last cell of each arithmetic constraint subblock, numbers in this subblock must satisfy the specified constraint.
- Succeeds if all cells are filled; fails otherwise
- Repeat enumerating for next possible solutions